Chapter 12

Uncoupled Converter and Extra Element Theorem

Pulsewidth Modulated Dc-to-Dc Power Conversion. By Byungcho Choi

Chapter Outline

DC Power Distribution System

Uncoupled Converter

Power Stage Dynamics of Uncoupled Converter

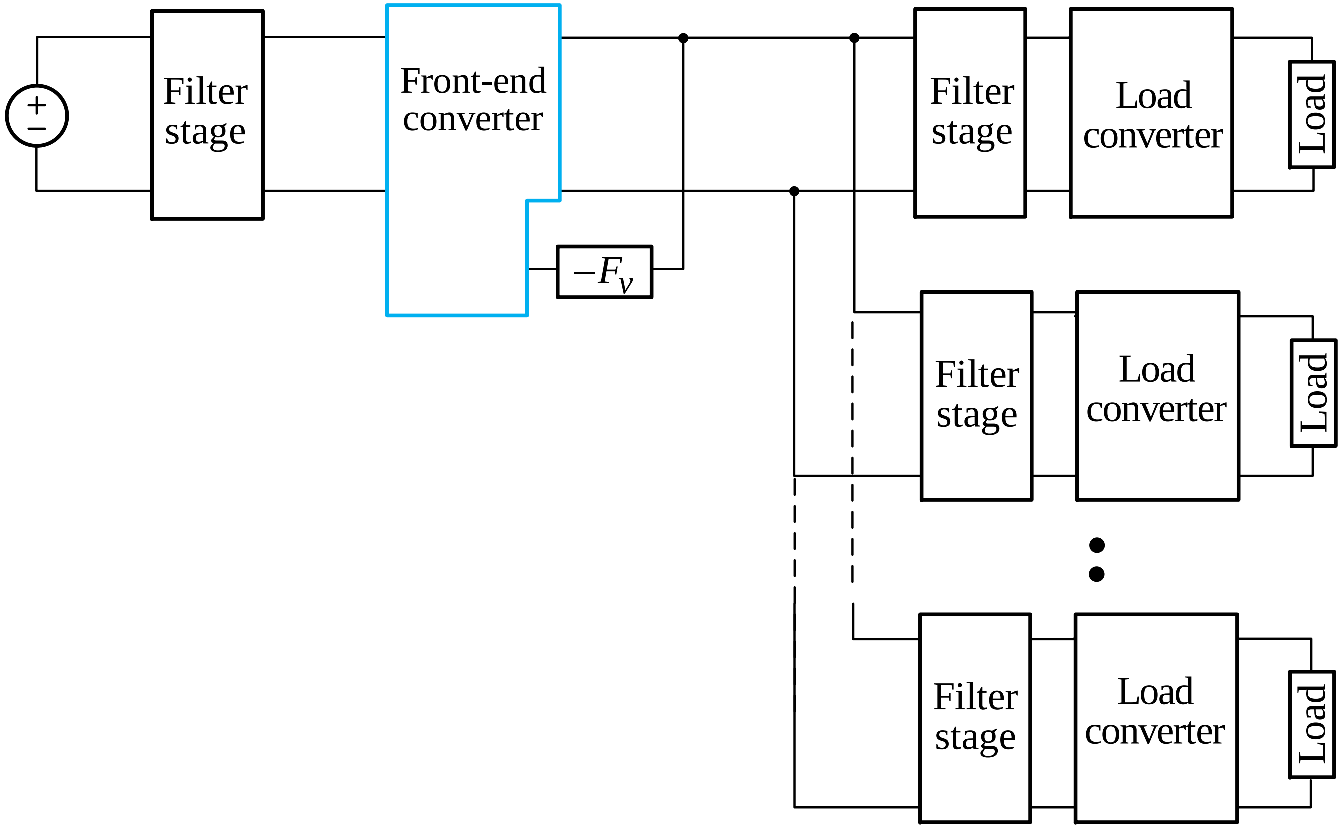
Control Design of Uncoupled Converter

Coupled Converters and Middlebrook's Extra Element Theorem

- Load-Coupled Converter
- Source-Coupled Converter

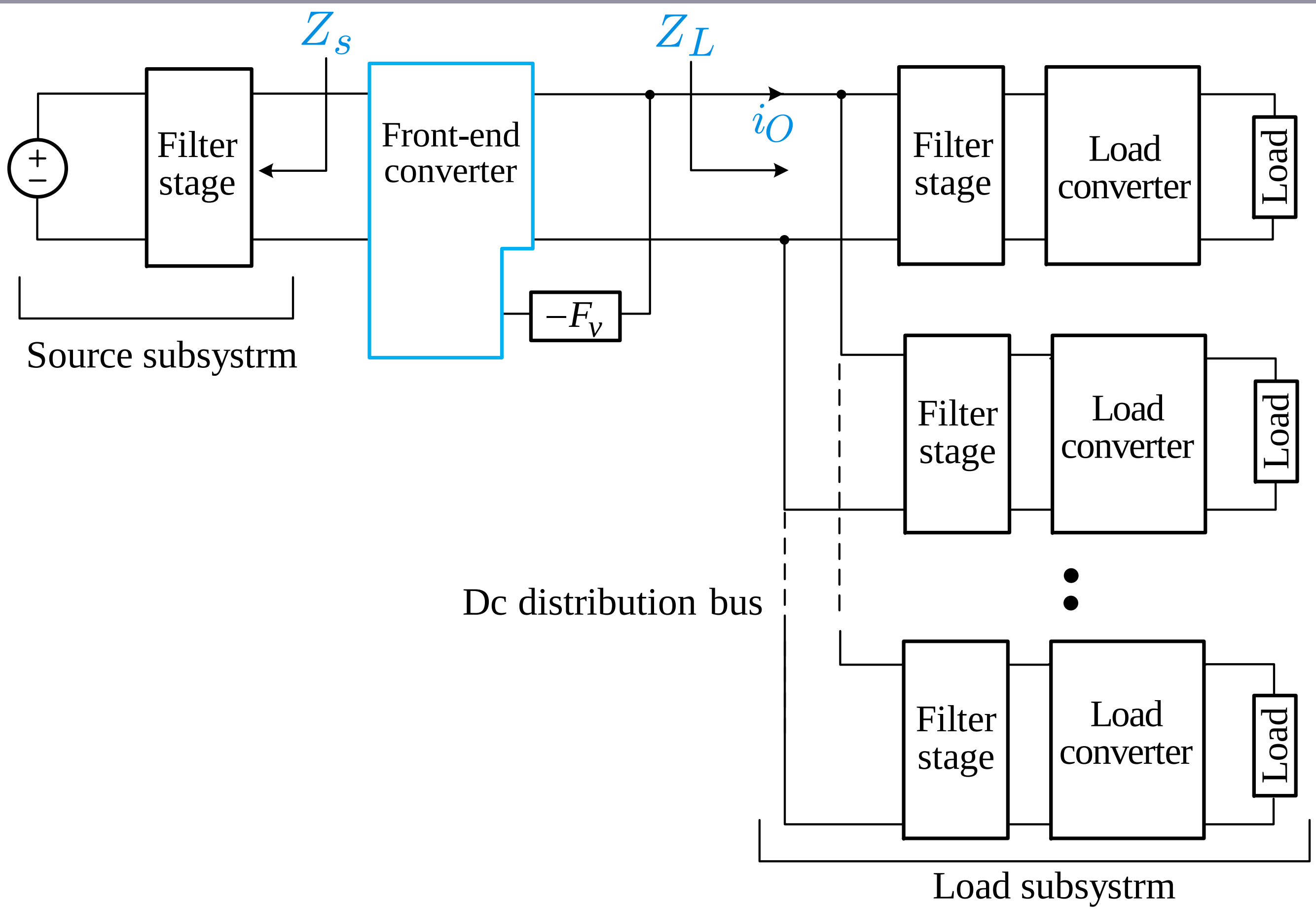
Middlebrook's Feedback Theorem

DC Power Distribution System for Computers



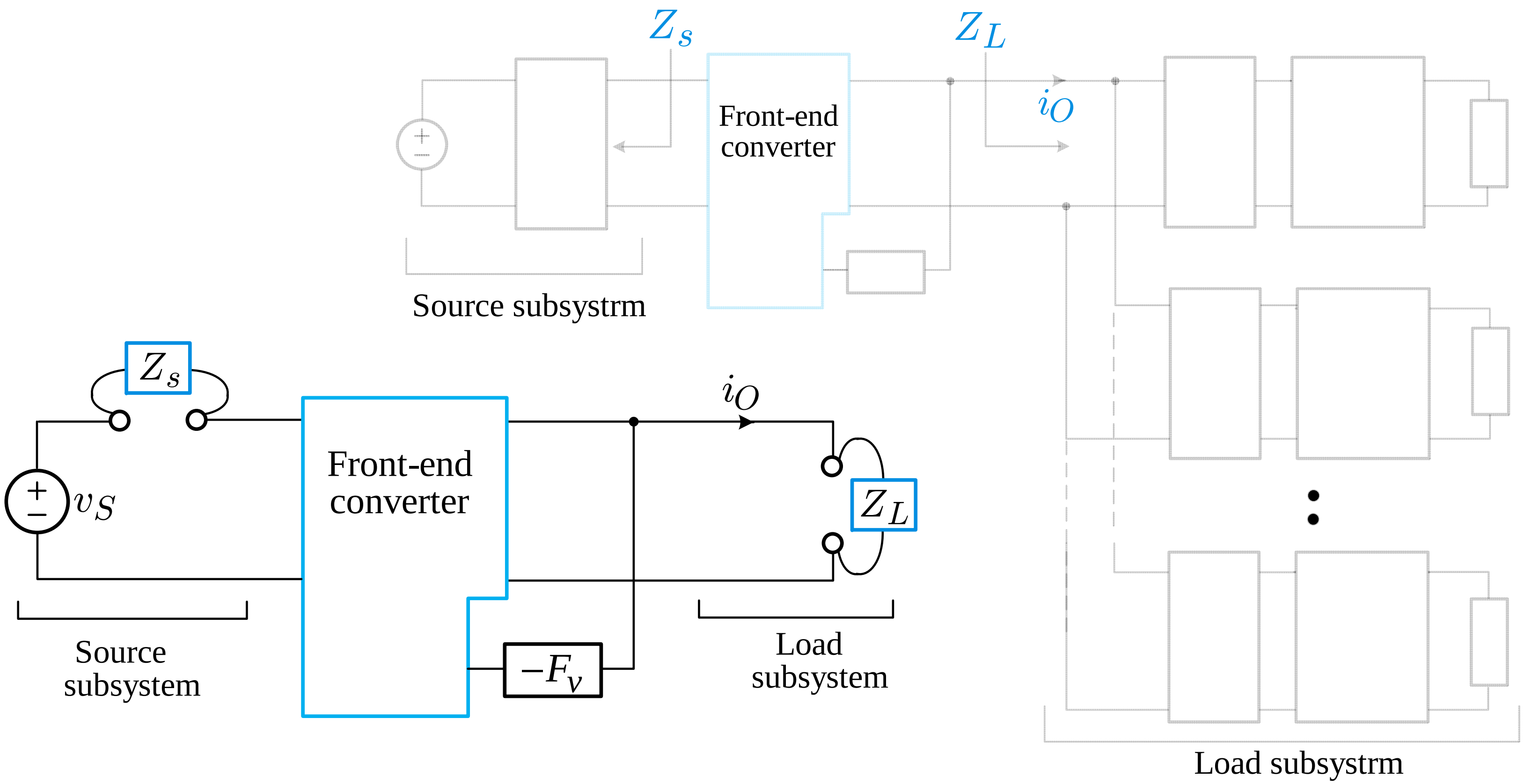
- Cascaded and paralled converters and filter stages for efficient and reliable power conversion
- Intermediate line filters to meet regulatory EMI standards
- Separate filter stage for each converter

DC Power Distribution System for Computers



- $Z_s(s)$: source impedance or output impedance of source subsystem
- $Z_L(s)$: load impedance or input impedance of load subsystem
- i_O: output current

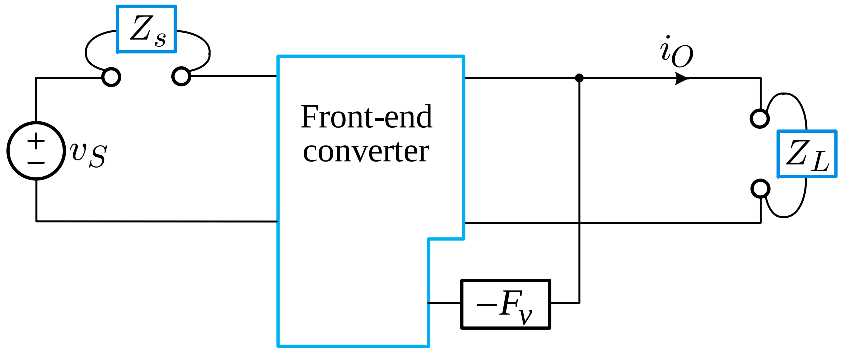
Equivalent Representation of Front-End Converter



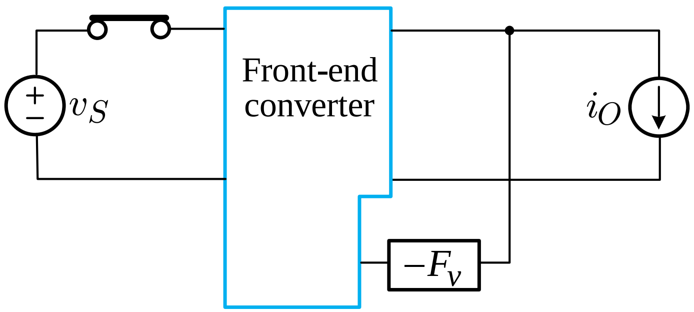
- v_S and i_O are always known in advance.
- $Z_s(s)$ and $Z_L(s)$ are unknown or undefined at the design stage of the converter.
- Design should be performed without any knowledge about $Z_s(s)$ and $Z_L(s)$.

Uncoupled Converter

- Equivalent representation of converter



- Uncoupled converter



$Z_s(s) = 0$: v_S is an ideal voltage source

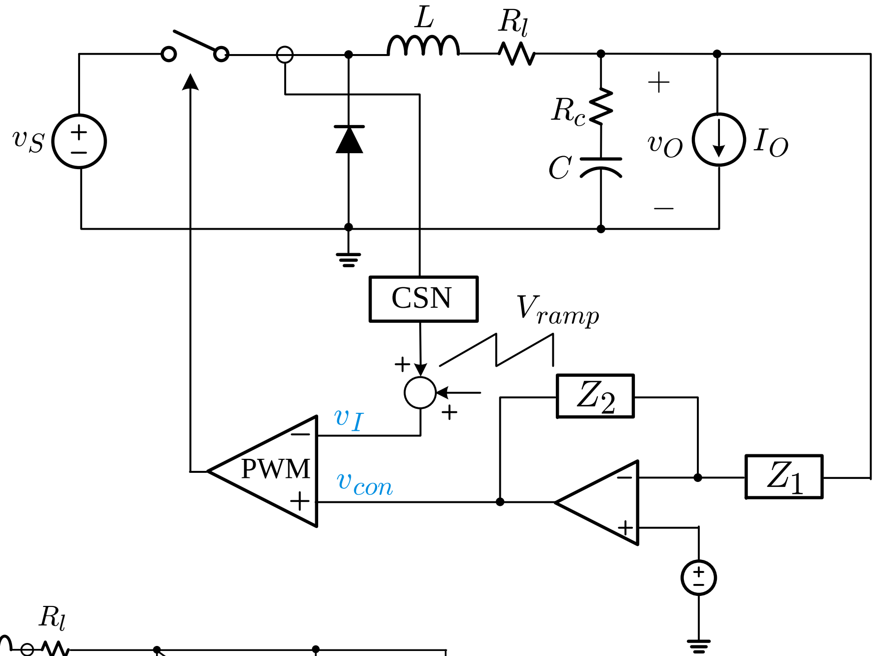
$Z_L(s) = \infty$: i_O is an ideal current sink

The control can be designed independently from the unknown $Z_s(s)$ and $Z_L(s)$.

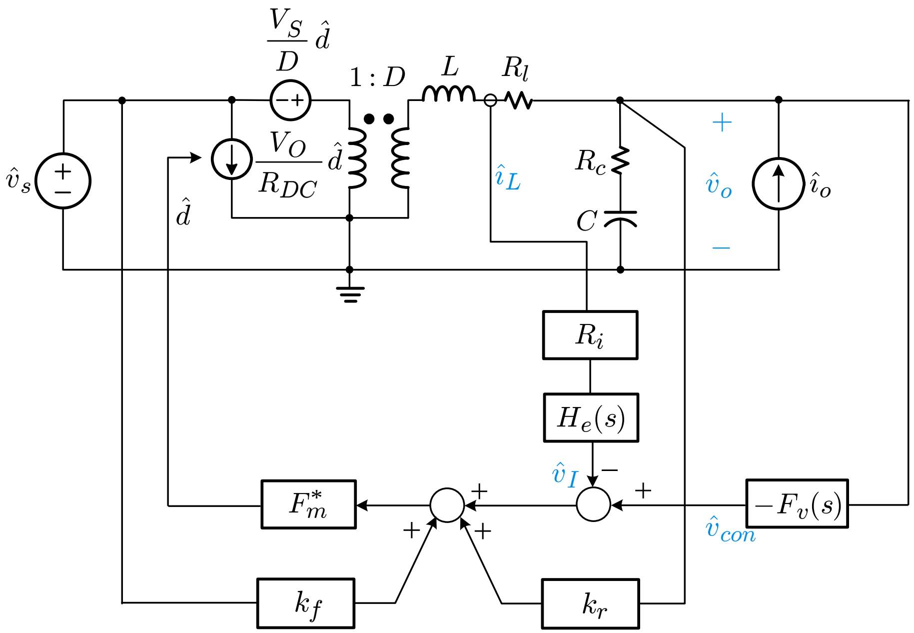
The converter performance can be evaluated with an ideal voltage source and current sink.

Whenever the information about $Z_s(s)$ and $Z_L(s)$ is available, the converter performance can be analyzed using the Extra Element Theorem.

Uncoupled Buck Converter

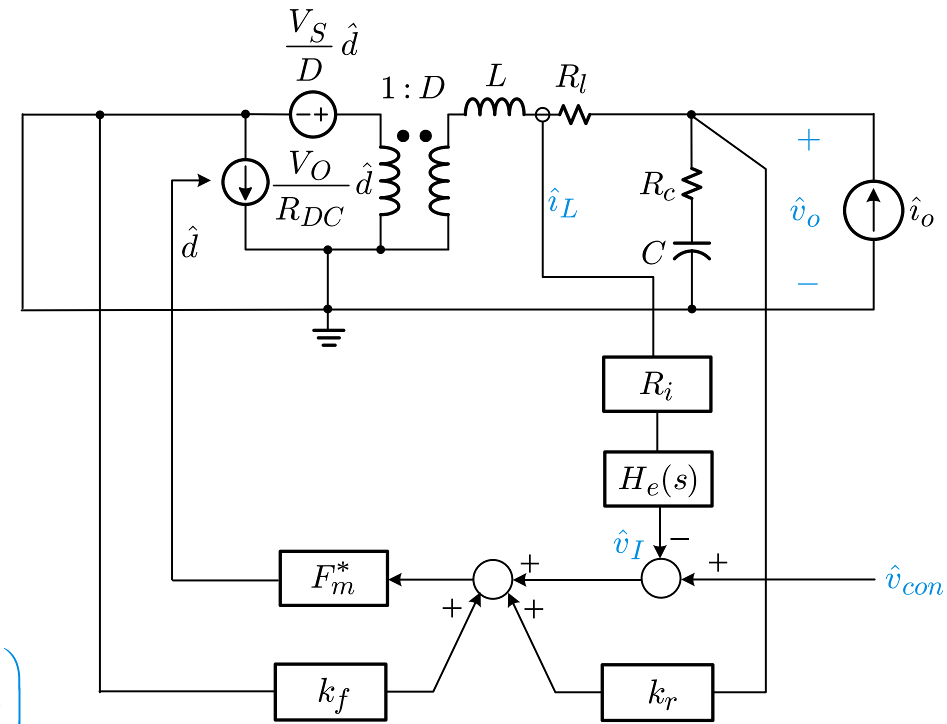


- Small-signal model



$$R_{DC} = \frac{V_O}{I_O} : \text{DC load parameter}$$

Control-to-Output Transfer Function



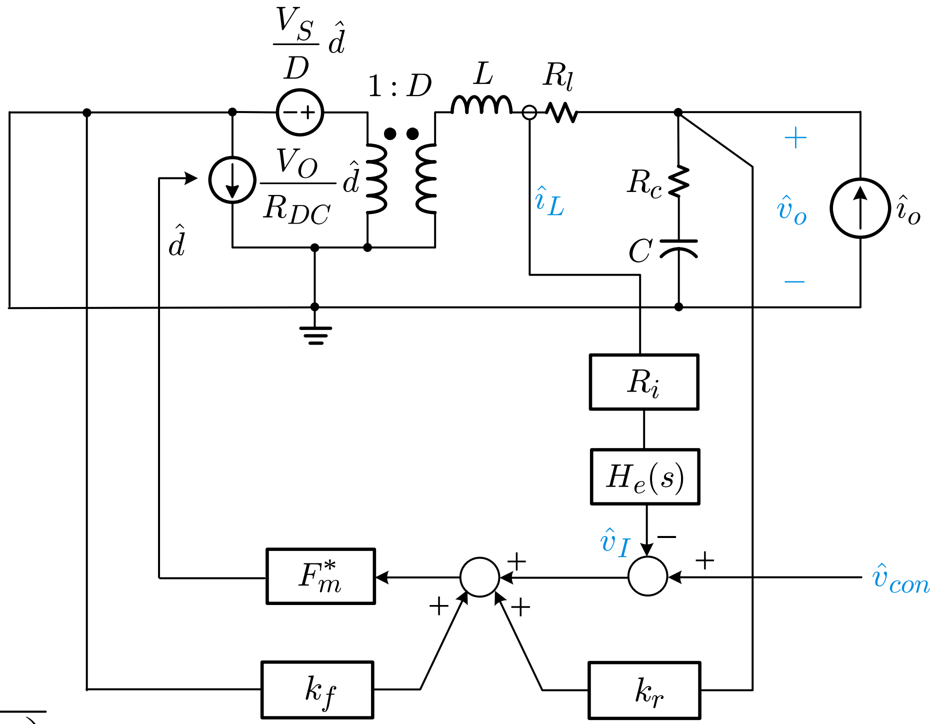
- $G_{vci}(s)=\dfrac{\hat{v}_o}{\hat{v}_{con}}\approx K_{vc}\dfrac{\left(1-\dfrac{s}{\omega_{rhp}}\right)\left(1+\dfrac{s}{\omega_{esr}}\right)}{\left(1+\dfrac{s}{\omega_{pl}}\right)\left(1+\dfrac{s}{Q_p\omega_n}+\dfrac{s^2}{\omega_n^2}\right)}$

$$K_{vc}=\frac{L}{R_i}\frac{1}{T_s(m_cD'-0.5)} \qquad \omega_{pl}=\frac{T_s(m_cD'-0.5)}{LC} \quad \text{with } m_c=1+\frac{S_e}{S_n}$$

Other parameters are the same as the case of a resistive load.

- The DC load parameter R_{DC} does not appear in the transfer function.

Current Loop Design



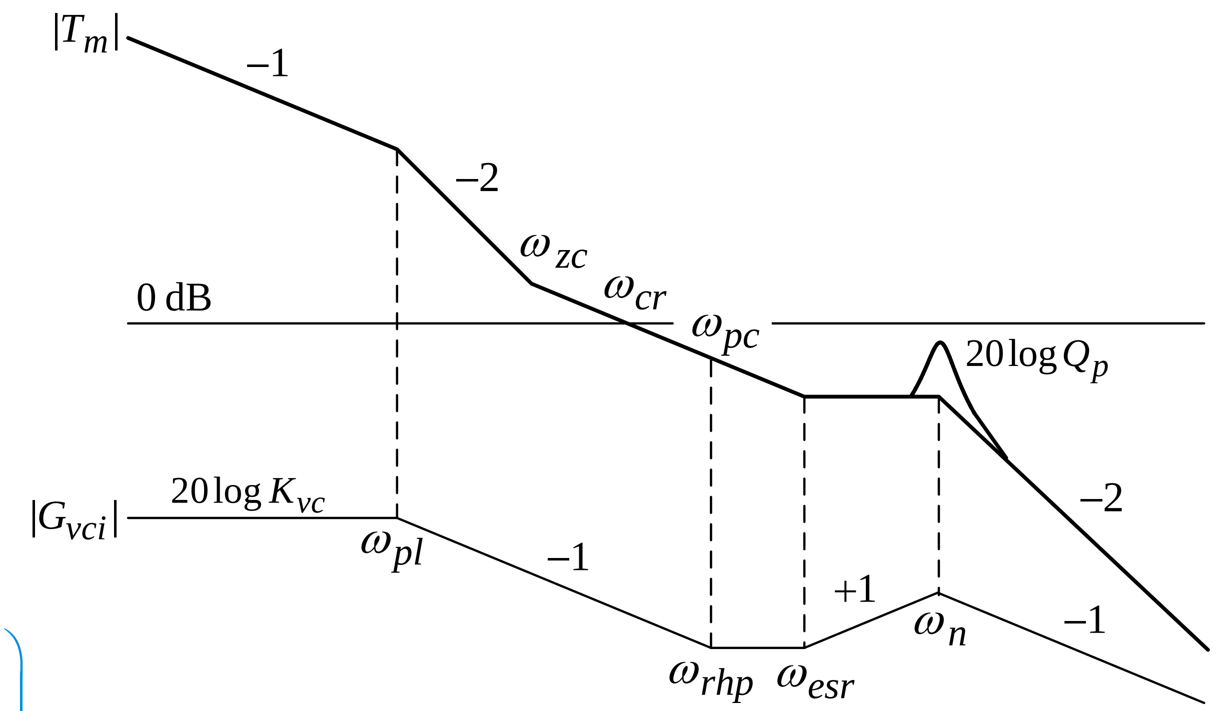
$$G_{vci}(s) = \frac{\hat{v}_o}{\hat{v}_{con}} \approx K_{vc} \frac{\left(1 - \frac{s}{\omega_{rhp}}\right)\left(1 + \frac{s}{\omega_{esr}}\right)}{\left(1 + \frac{s}{\omega_{pl}}\right)\left(1 + \frac{s}{Q_p \omega_n} + \frac{s^2}{\omega_n^2}\right)}$$

- Current loop design remains the same as the resistor load case.

$$0.3 \; < \; Q_p = \frac{1}{\pi\left(\left(1 + \frac{S_e}{S_n}\right)D' - 0.5\right)} \; < \; 1.3$$

Control Design Procedures

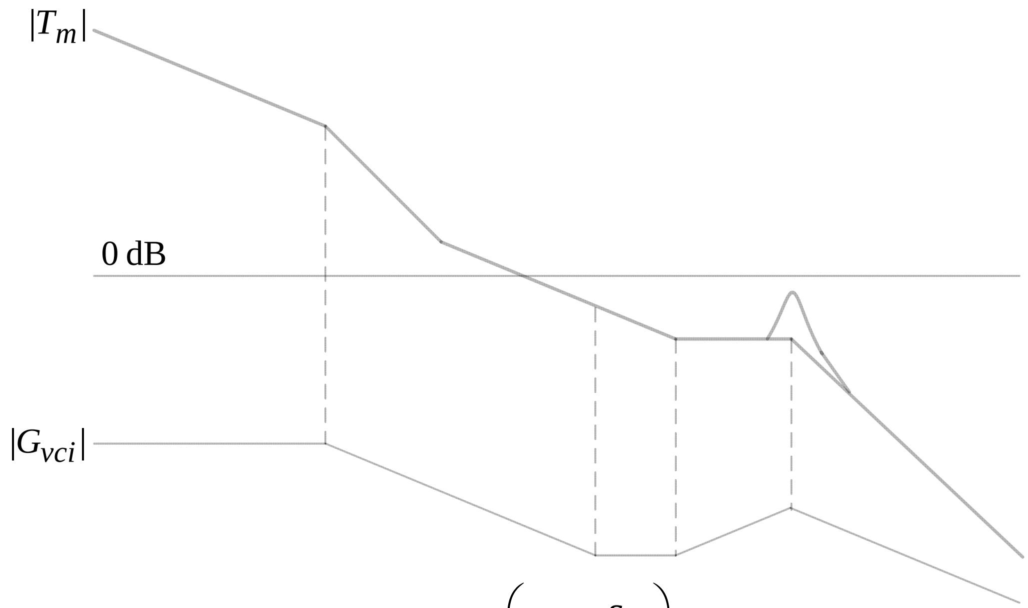
Voltage Loop Design



- $$F_v(s) = \frac{K_v\left(1+\dfrac{s}{\omega_{zc}}\right)}{s\left(1+\dfrac{s}{\omega_{pc}}\right)}$$

- $$T_m(s) = G_{vci}(s)F_v(s) = K_{vc}\frac{\left(1-\dfrac{s}{\omega_{rhp}}\right)\left(1+\dfrac{s}{\omega_{esr}}\right)}{\left(1+\dfrac{s}{\omega_{pl}}\right)\left(1+\dfrac{s}{Q_p\omega_n}+\dfrac{s^2}{\omega_n^2}\right)}\frac{K_v\left(1+\dfrac{s}{\omega_{zc}}\right)}{s\left(1+\dfrac{s}{\omega_{pc}}\right)}$$

Voltage Feedback Compensation



- $$T_m(s) = K_{vc}\frac{\left(1-\frac{s}{\omega_{rhp}}\right)\left(1+\frac{s}{\omega_{esr}}\right)}{\left(1+\frac{s}{\omega_{pl}}\right)\left(1+\frac{s}{Q_p\omega_n}+\frac{s^2}{\omega_n^2}\right)}F_v(s) \quad \text{with} \quad F_v(s) = \frac{K_v\left(1+\frac{s}{\omega_{zc}}\right)}{s\left(1+\frac{s}{\omega_{pc}}\right))}$$

- Selections of $F_v(s)$ parameters is the same as the case of a resistive load.

$$\omega_{pc} = \min\{\omega_{rhp}\ \omega_{esr}\ \omega_s/2\} \quad \omega_{zc} = (0.6-0.8)\ \omega_o \quad K_v = \frac{\omega_{zc}\,\omega_{cr}}{K_{vc}\omega_{pl}}$$

- Voltage loop design becomes the same provided that the product $K_{vc}\omega_{pl}$ remains unchanged.

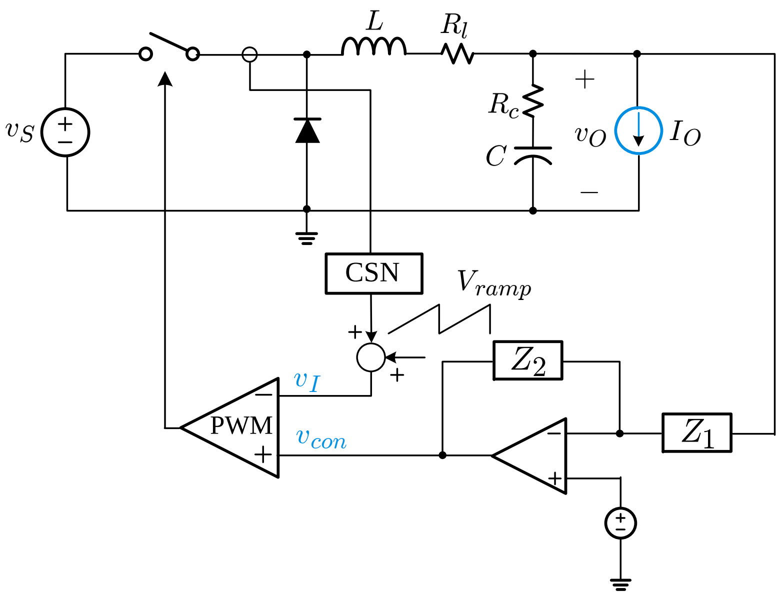
$K_{vc}\,\omega_{pl}$ Product Comparison

$$G_{vci}(s) \approx K_{vs} \frac{\left(1-\dfrac{s}{\omega_{rhp}}\right)\left(1+\dfrac{s}{\omega_{esr}}\right)}{\left(1+\dfrac{s}{\omega_{pl}}\right)\left(1+\dfrac{s}{Q_p\omega_n}+\dfrac{s^2}{\omega_n^2}\right)}$$

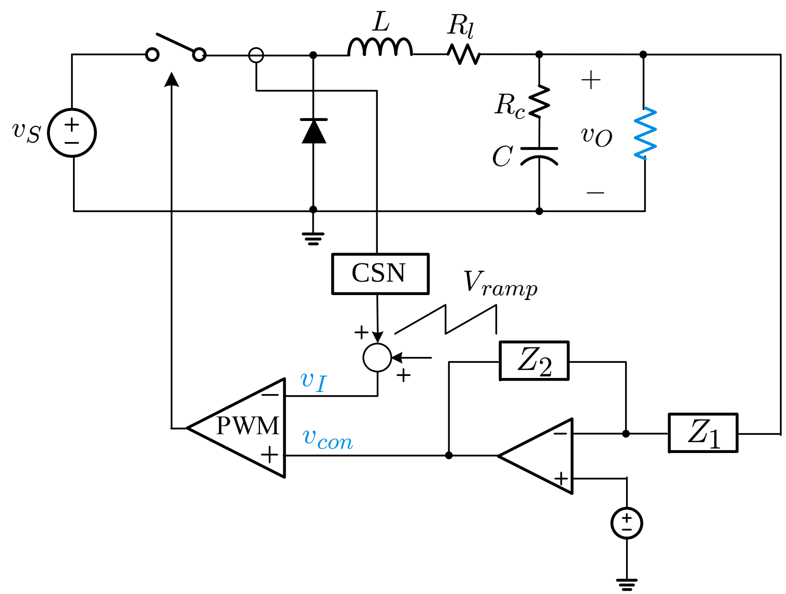
	Uncoupled converter	Converter with resistive load
K_{vc}	$\frac{L}{R_i}\frac{1}{T_s(m_c D' - 0.5)}$	$\frac{R}{R_i}\frac{1}{1+\frac{RT_s}{L}(m_c D' - 0.5)}$
ω_{pl}	$\frac{T_s(m_c D' - 0.5)}{LC}$	$\frac{1}{CR}\left(1+\frac{RT_s}{L}(m_c D' - 0.5)\right)$
$K_{vc}\,\omega_{pl}$	$\frac{1}{R_i C}$	$\frac{1}{R_i C}$

Control Design Procedures

Control Design Summary



Uncoupled buck converter

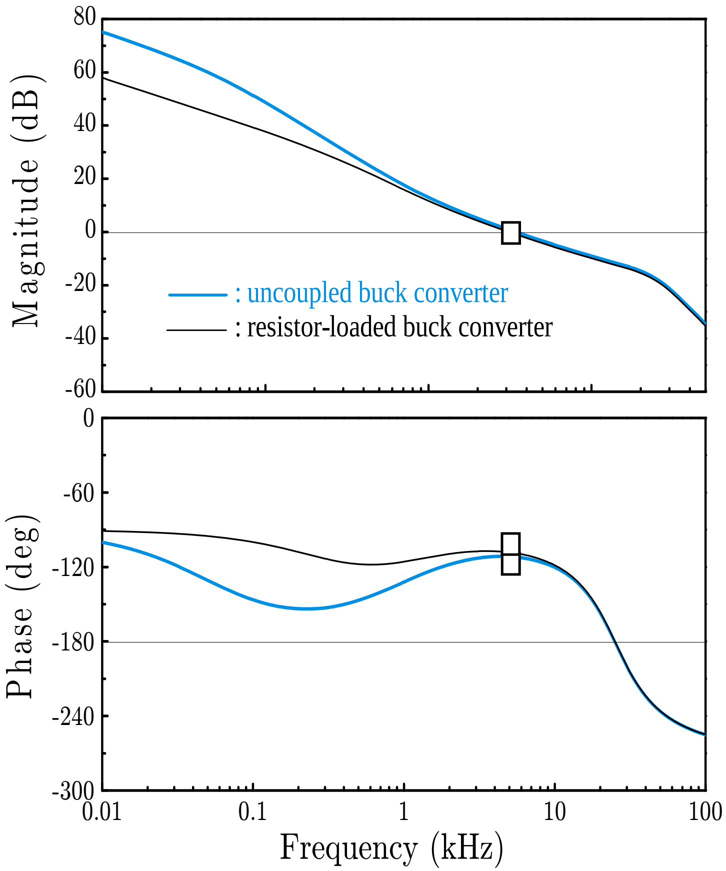


Resisor-coupled buck converter

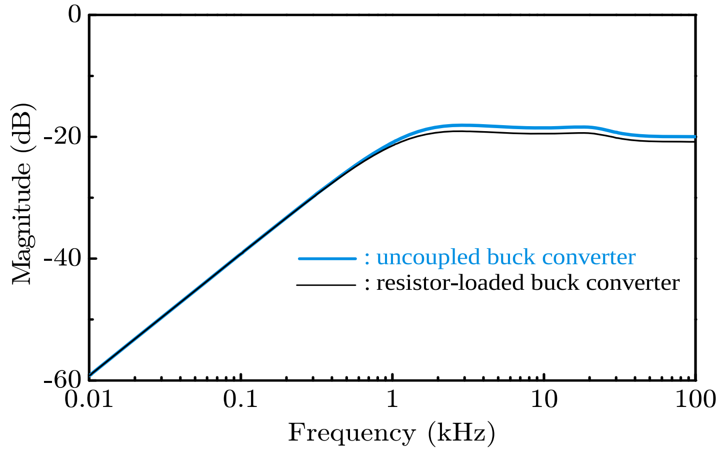
- Control design of the uncoupled converter remains the same as the resistor-loaded converter.
- Standard design procedures for resistive loads are adoptable to uncoupled converters.
- Conclusions can be extended to other onverter topologies.

Converter Performance

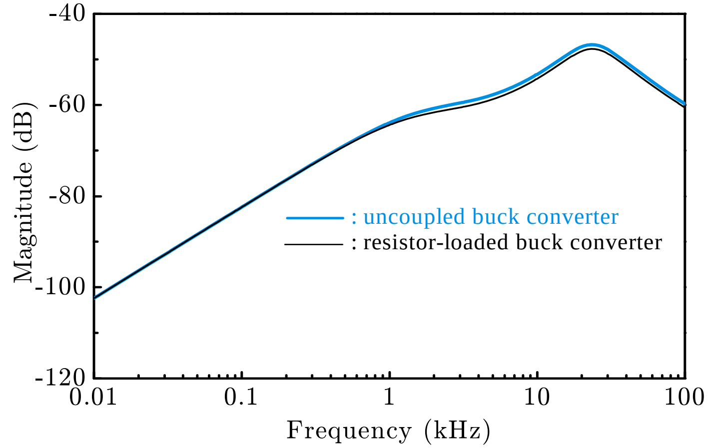
Loop gain



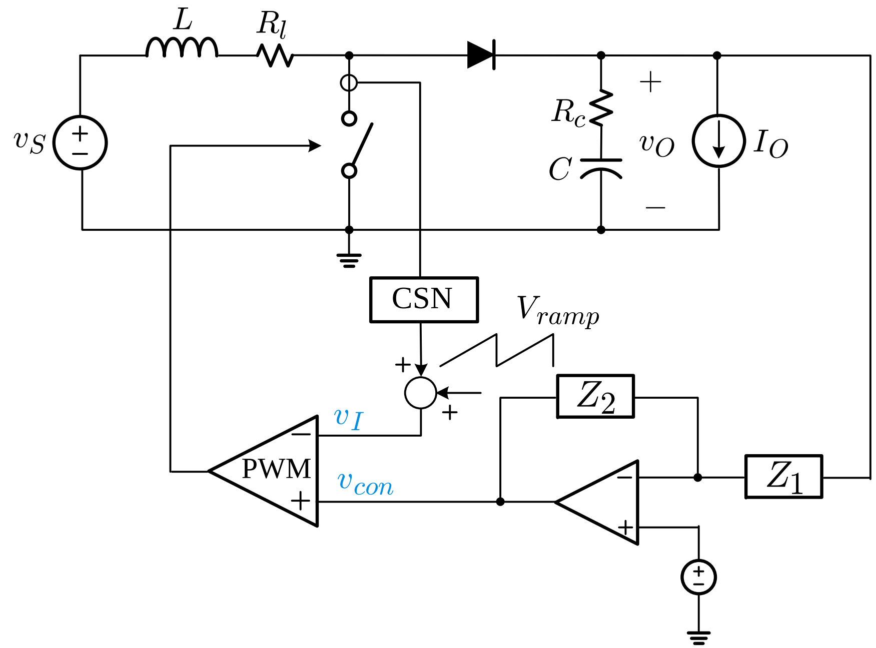
Output impedance



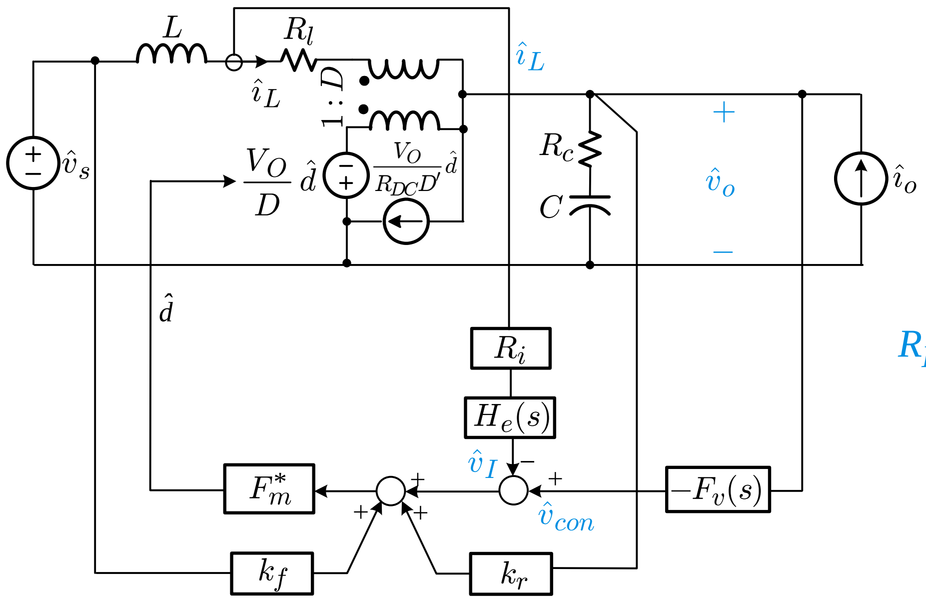
Audio-susceptibility



Uncoupled Boost Converter

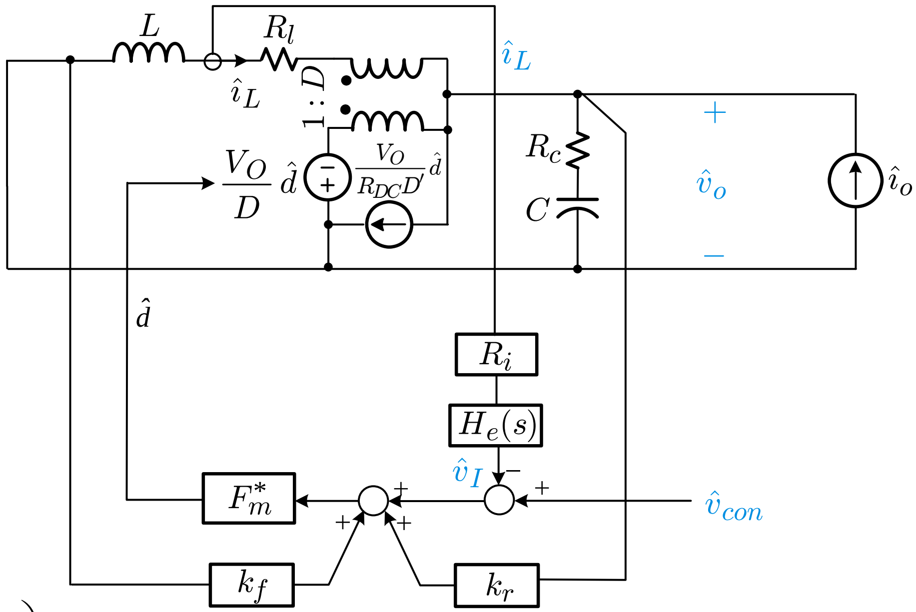


- Small-signal model



$R_{DC} = \dfrac{V_O}{I_O}$: DC load parameter

Control-to-Output Transfer Function



- $G_{vci}(s) = \dfrac{\hat{v}_o}{\hat{v}_{con}} \approx K_{vc} \dfrac{\left(1 - \dfrac{s}{\omega_{rhp}}\right)\left(1 + \dfrac{s}{\omega_{esr}}\right)}{\left(1 + \dfrac{s}{\omega_{pl}}\right)\left(1 + \dfrac{s}{Q_p \omega_n} + \dfrac{s^2}{\omega_n^2}\right)}$

$$K_{vc} = \frac{L}{R_i} \frac{1}{T_s D'^2 (m_c D' - 0.5) + \dfrac{L}{R_{DC} D'}} \qquad \omega_{pl} = \frac{T_s D'^2 (m_c D' - 0.5) + \dfrac{L}{R_{DC} D'}}{\dfrac{LC}{D'}} \qquad \omega_{rhp} = \frac{R_{DC} D'^2}{L}$$

- The DC load parameter R_{DC} is a key parameter.

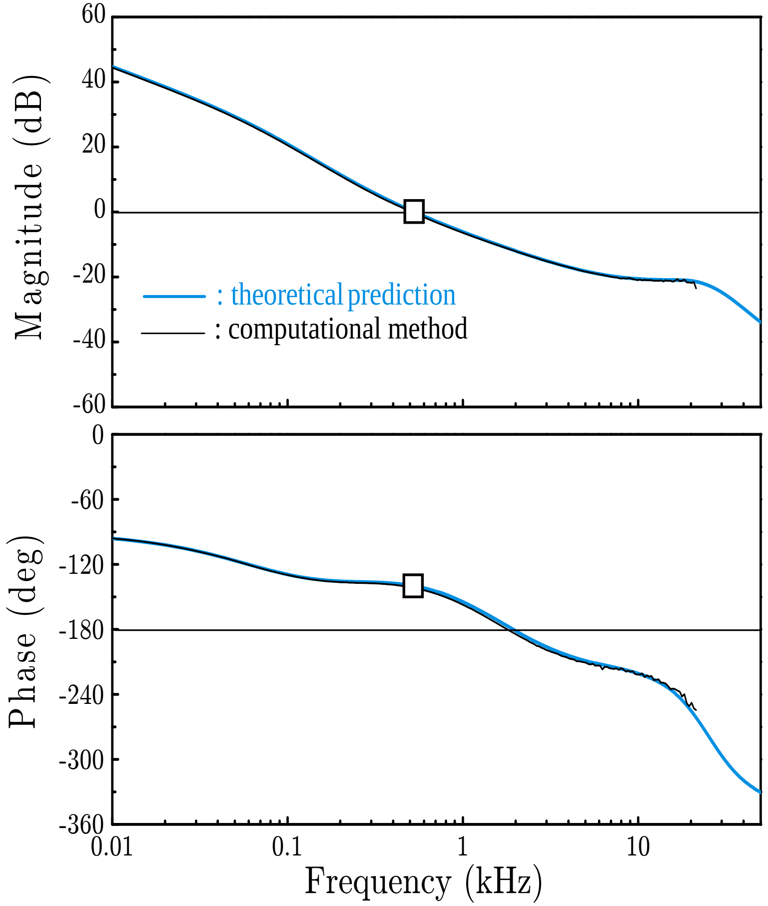
$K_{vc}\,\omega_{pl}$ Product Comparison

$$G_{vci}(s) \approx K_{vs}\frac{\left(1-\frac{s}{\omega_{rhp}}\right)\left(1+\frac{s}{\omega_{esr}}\right)}{\left(1+\frac{s}{\omega_{pl}}\right)\left(1+\frac{s}{Q_p\omega_n}+\frac{s^2}{\omega_n^2}\right)}$$

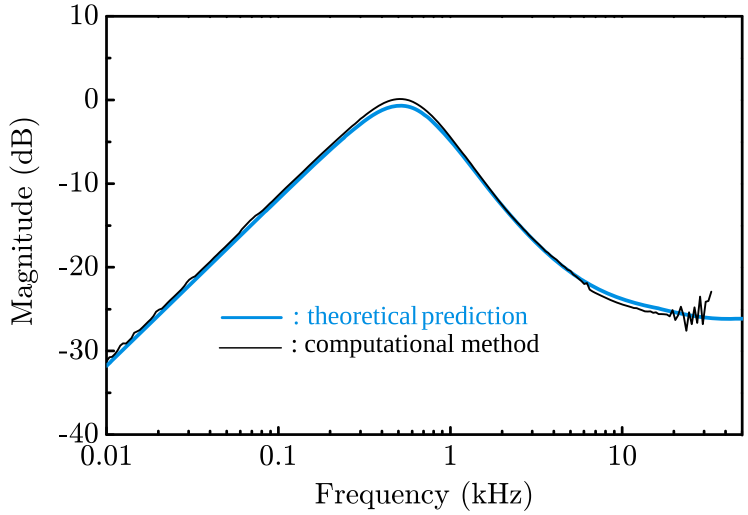
	Uncoupled converter	Converter with resistive load
K_{vc}	$\frac{L}{R_i}\frac{1}{T_sD'^2(m_c-0.5)+\frac{L}{R_{DC}D'}}$	$\frac{L}{R_i}\frac{1}{T_sD'^2(m_c-0.5)+\frac{2L}{R\,D'}}$
ω_{pl}	$\frac{T_sD'^2(m_c-0.5)+\frac{L}{R_{DC}D'}}{\frac{LC}{D'}}$	$\frac{T_sD'^2(m_c-0.5)+\frac{2L}{R\,D'}}{\frac{LC}{D'}}$
$K_{vc}\,\omega_{pl}$	$\frac{D'}{R_iC}$	$\frac{D'}{R_iC}$

Converter Performance

Loop gain



Output impedance

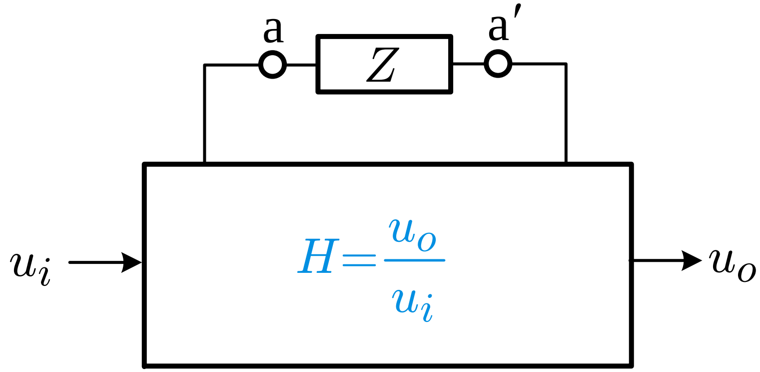


Buck/Boost Converter

$$G_{vci}(s) \approx K_{vs} \frac{\left(1 - \frac{s}{\omega_{rhp}}\right)\left(1 + \frac{s}{\omega_{esr}}\right)}{\left(1 + \frac{s}{\omega_{pl}}\right)\left(1 + \frac{s}{Q_p \omega_n} + \frac{s^2}{\omega_n^2}\right)}$$

	Uncoupled converter	Converter with resistive load
K_{vc}	$\frac{L}{R_i} \frac{1}{T_s D'^2 (m_c - 0.5) + \frac{DL}{R_{DC} D'}}$	$\frac{L}{R_i} \frac{1}{T_s D'^2 (m_c - 0.5) + \frac{(1+D)L}{R D'}}$
ω_{pl}	$\frac{T_s D'^2 (m_c - 0.5) + \frac{DL}{R_{DC} D'}}{\frac{LC}{D'}}$	$\frac{T_s D'^2 (m_c - 0.5) + \frac{(1+D)L}{R D'}}{\frac{LC}{D'}}$
$K_{vc}\, \omega_{pl}$	$\frac{D'}{R_i C}$	$\frac{D'}{R_i C}$

Middlebrook's Extra Element Theorem



- Definitions

  $H(s) = \dfrac{u_o(s)}{u_i(s)}$: transfer gain of interest

  $Z(s)$: impedance of the circuit component which is desifnated as the extra element

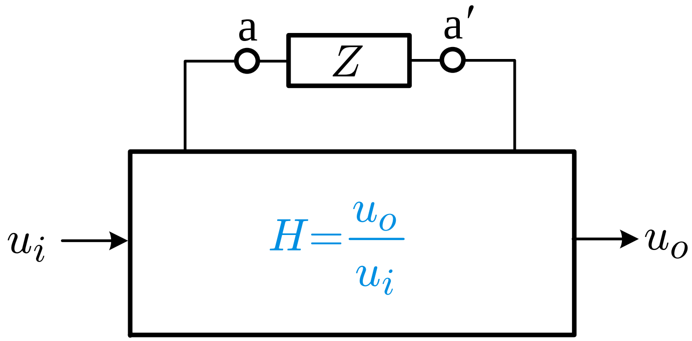
- The Extra Element Theorem: $H(s) = H_\infty(s)\dfrac{1+\dfrac{\bar{\varsigma}(s)}{Z(s)}}{1+\dfrac{\bar{z}(s)}{Z(s)}}$

  $H_\infty(s)$: transfer gain $H(s)$ evaluated with the extra element removed, denoted as the open-circut transfer gain

  $\bar{z}(s)$: input impedance looking into a-a′ with the input variable $u_i(s)$ disabled, denoted as the driving point impedance

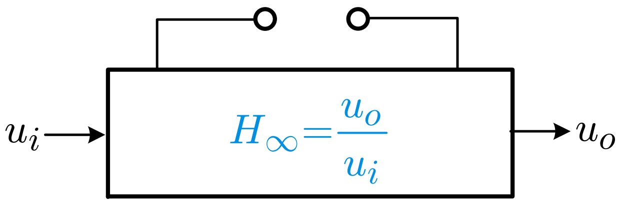
  $\bar{\varsigma}(s)$: input impedance looking into a-a′ with the input variable $u_o(s)$ nullified, denoted as the null driving point impedance

Pictorial Illumination



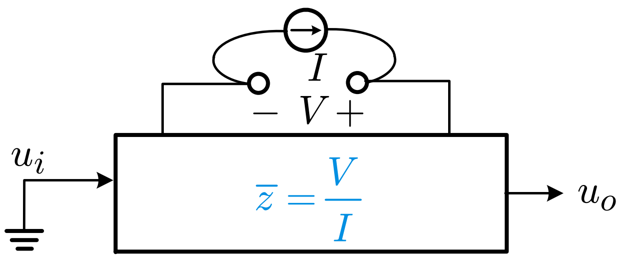
$$H(s) = H_\infty(s) \frac{1 + \frac{\bar{\varsigma}(s)}{Z(s)}}{1 + \frac{\bar{z}(s)}{Z(s)}}$$

$H_\infty(s)$:



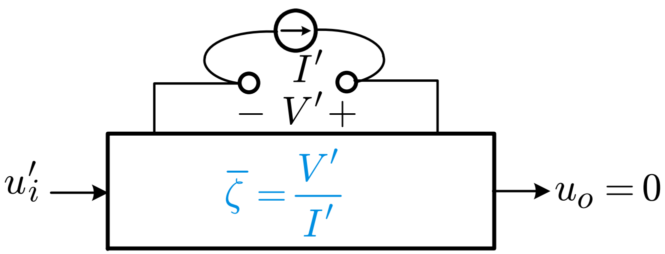
$H_\infty(s) = u_o(s) / u_i(s)$ with $Z(s) = \infty$

$\bar{z}(s)$:



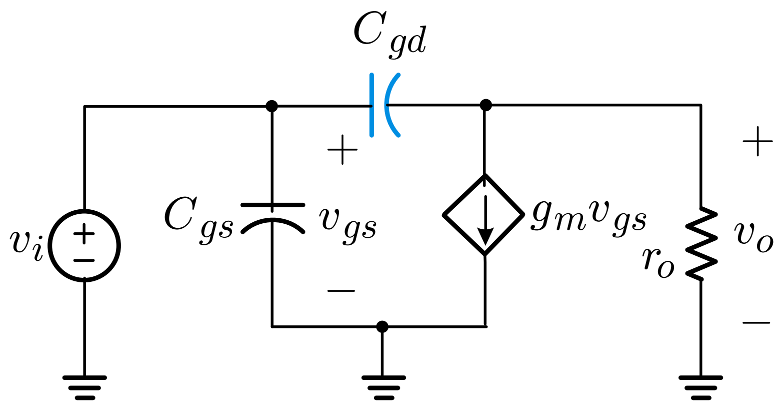
$\bar{z}(s) = V(s) / I(s)$ with $u_i(s) = 0$

$\bar{\varsigma}(s)$:

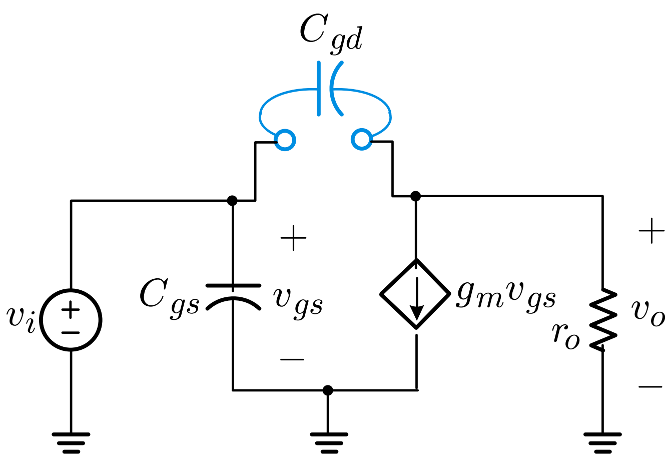


$\bar{\varsigma}(s) = V'(s) / I'(s)$ with $u_o(s) = 0$

Application to MOSFET Amplifier



- Evaluate the voltage gain $\dfrac{v_o(s)}{v_i(s)}$ using EET
- Consider C_{gd} as the extra element

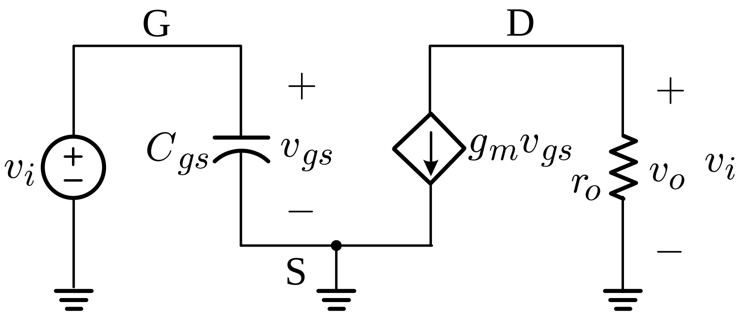


$$H(s) = \frac{v_o(s)}{v_i(s)} = H_\infty(s)\frac{1+\dfrac{\overline{\varsigma}(s)}{Z(s)}}{1+\dfrac{\overline{z}(s)}{Z(s)}}$$

with $Z(s) = \dfrac{1}{sC_{gd}}$

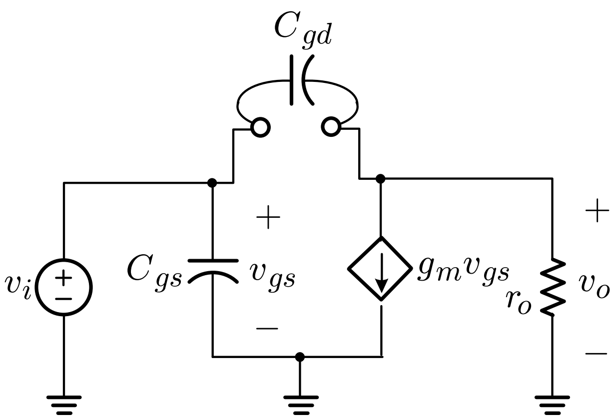
Application to MOSFET Amplifier

- MOSFET amplifier with $C_{gd} = 0 \Rightarrow Z(s) = \dfrac{1}{sC_{gd}} = \infty$



$$H_\infty(s) = \left.\frac{v_o(s)}{v_i(s)}\right|_{Z(s)=\infty} = -g_m r_o$$

- MOSFET amplifier with $C_{gd} \neq 0 \Rightarrow Z(s) = \dfrac{1}{sC_{gd}} \neq \infty$

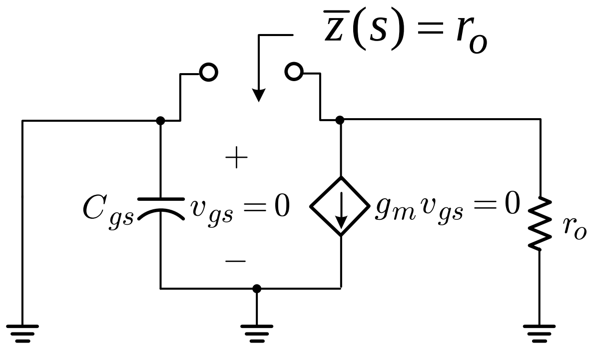


$$H(s) = \left.\frac{v_o(s)}{v_i(s)}\right|_{Z(s)\neq\infty} = H_\infty(s)\frac{1+\dfrac{\bar{\varsigma}(s)}{Z(s)}}{1+\dfrac{\bar{z}(s)}{Z(s)}}$$

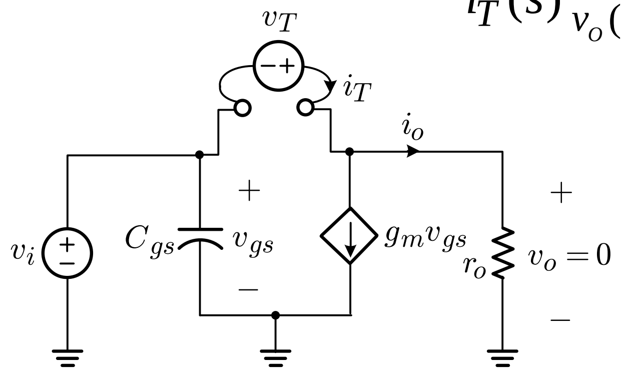
$$= -g_m r_o \frac{1+sC_{gd}\,\bar{\varsigma}(s)}{1+sC_{gd}\,\bar{z}(s)}$$

Application to MOSFET Amplifier

- Evaluation of $\overline{z}(s)$



- Evaluation of $\overline{\varsigma}(s) = \left.\dfrac{v_T(s)}{i_T(s)}\right|_{v_o(s)=0}$



i) $v_o(s) = 0 \quad \Rightarrow i_o = 0 \quad \Rightarrow i_T = g_m v_{gs}$

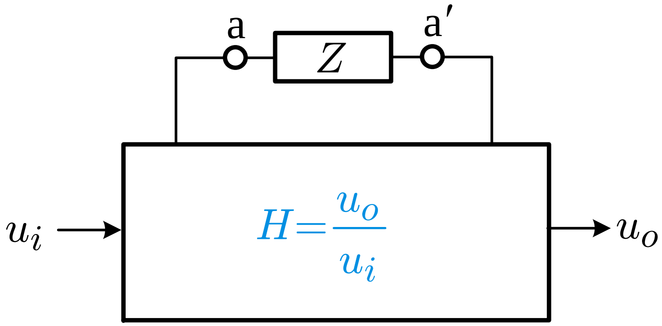
ii) $v_o(s) = 0 \quad \Rightarrow v_T = -v_{gs}$

$$\overline{\varsigma}(s) = \frac{v_T}{i_T} = \frac{-v_{gs}}{g_m v_{gs}} = -\frac{1}{g_m}$$

- Tranfer gain $H(s) = \left.\dfrac{v_o(s)}{v_i(s)}\right|_{Z(s)\neq\infty} = -g_m r_o \dfrac{1 + sC_{gd}\overline{\varsigma}(s)}{1 + sC_{gd}\overline{z}(s)}$

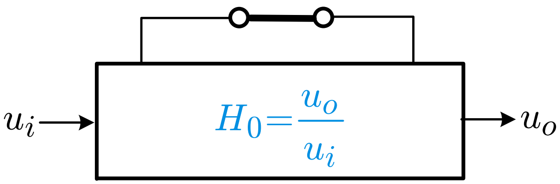
$$= -g_m r_o \frac{1 - \dfrac{sC_{gd}}{g_m}}{1 + sC_{gd} r_o}$$

Alternative Form of EET



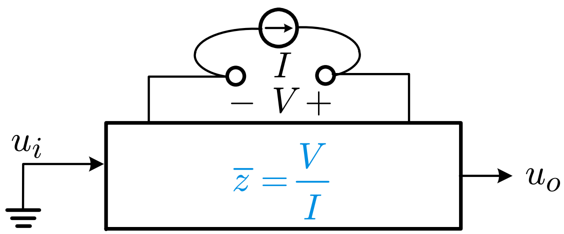
$$H(s) = H_0(s)\frac{1+\dfrac{Z(s)}{\overline{\varsigma}(s)}}{1+\dfrac{Z(s)}{\overline{z}(s)}}$$

$H_0(s)$:



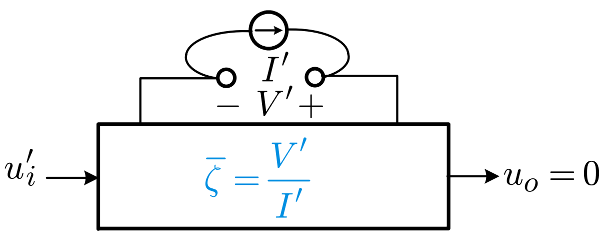
$H_0(s) = u_o(s) / u_i(s)$ with $Z(s)=0$

$\overline{z}(s)$:



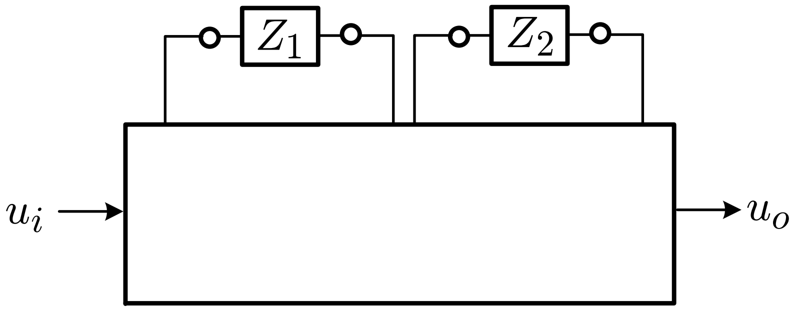
$\overline{z}(s) = V(s) / I(s)$ with $u_i(s)=0$

$\overline{\varsigma}(s)$:

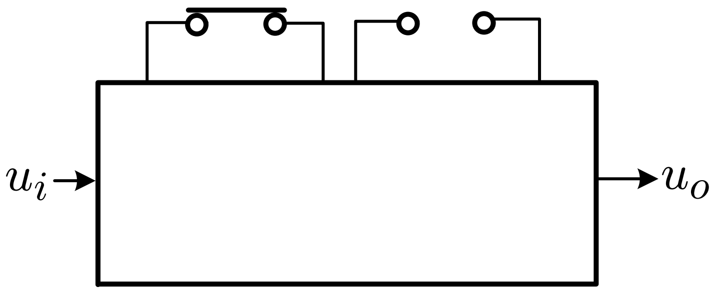


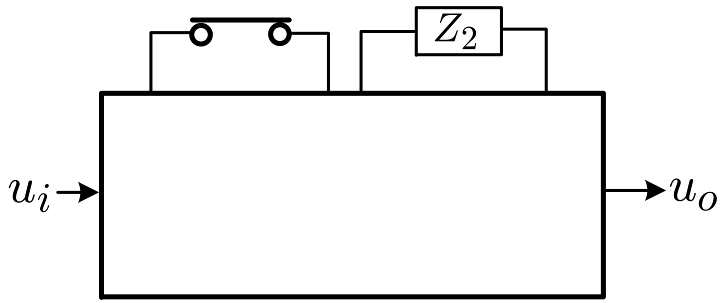
$\overline{\varsigma}(s) = V'(s) / I'(s)$ with $u_o(s)=0$

Extension to 2 EET



- First adoption of EET

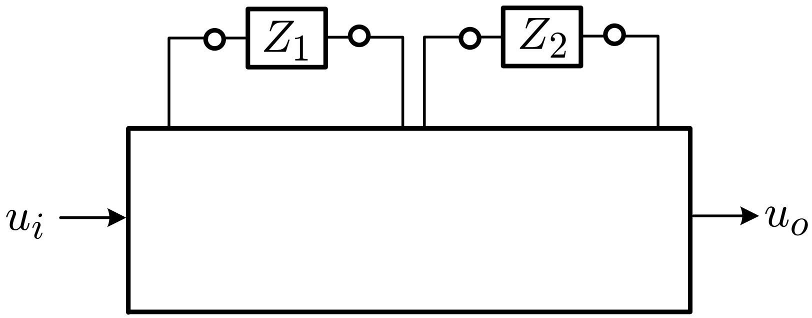




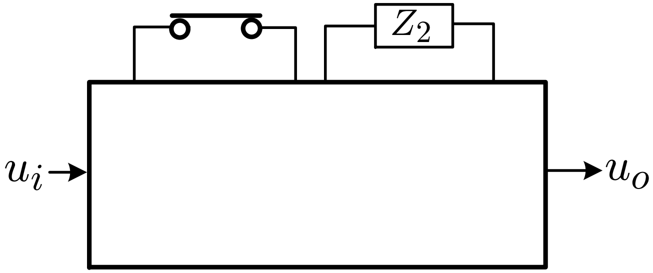
$$\left.\frac{u_o(s)}{u_i(s)}\right|_{Z_1(s)=0\ Z_2(s)=\infty} = H_s(s)$$

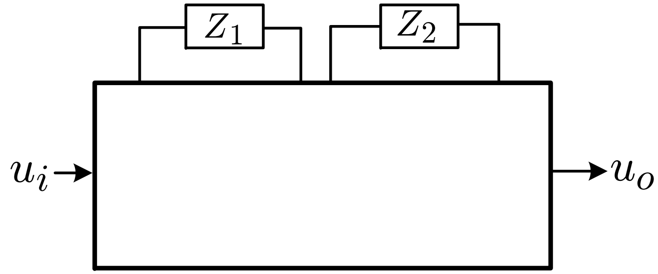
$$\left.\frac{u_o(s)}{u_i(s)}\right|_{\substack{Z_1(s)=0\\ Z_2(s)\neq\infty}} = H_s(s)\frac{1+\dfrac{\bar{\varsigma}_2(s)}{Z_2(s)}}{1+\dfrac{\bar{z}_2(s)}{Z_2(s)}}$$

Extension of EET- 2 EET



- Second adoption of EET



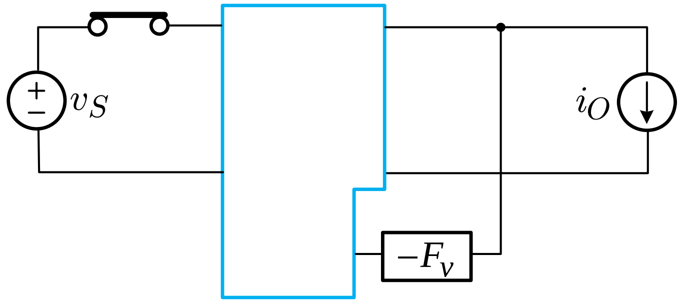


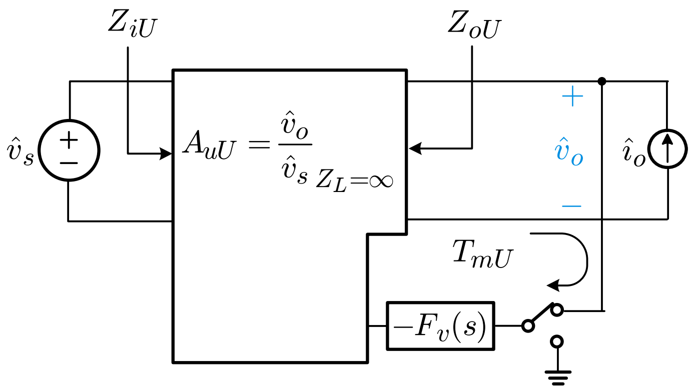
$$\left.\frac{u_o(s)}{u_i(s)}\right|_{\substack{Z_1(s)=0\\ Z_2(s)\neq\infty}} = H_s(s)\frac{1+\dfrac{\bar{\varsigma}_2(s)}{Z_2(s)}}{1+\dfrac{\bar{z}_2(s)}{Z_2(s)}}$$

$$\left.\frac{u_o(s)}{u_i(s)}\right|_{\substack{Z_1(s)\neq 0\\ Z_2(s)\neq\infty}} = \left(H_s(s)\frac{1+\dfrac{\bar{\varsigma}_2(s)}{Z_2(s)}}{1+\dfrac{\bar{z}_2(s)}{Z_2(s)}}\right)\left(\frac{1+\dfrac{Z_1(s)}{\bar{\varsigma}_1(s)}}{1+\dfrac{Z_1(s)}{\bar{z}_1(s)}}\right)$$

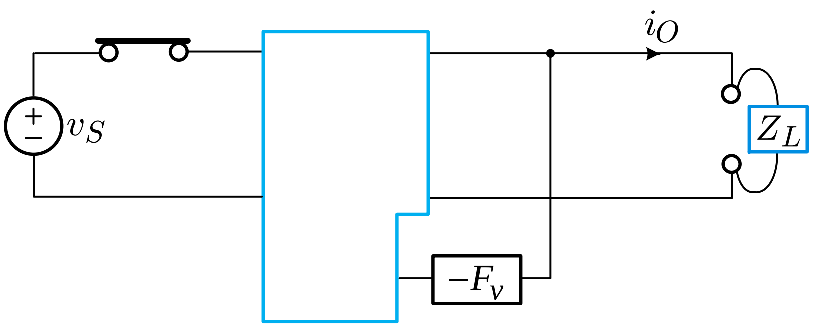
Load-Coupled Converter

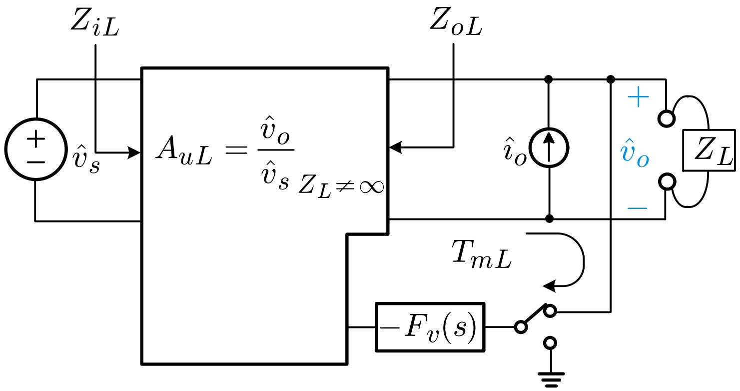
- Uncoupled converter



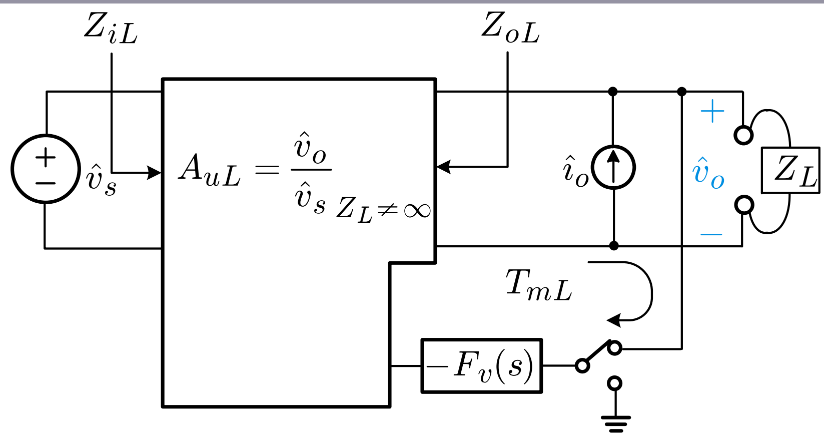


- Load-coupled converter





Performance of Load-Coupled Converter

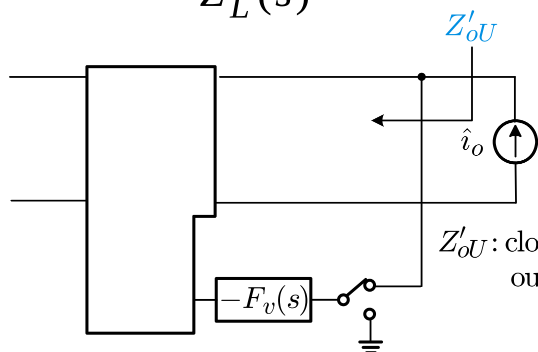


$$A_{uL}(s) = A_{uU}(s)\frac{1}{1+\dfrac{Z_{oU}(s)}{Z_L(s)}}$$

$$T_{mL}(s) = T_{mU}(s)\frac{1}{1+\left(1+T_{mU}(s)\right)\dfrac{Z_{oU}(s)}{Z_L(s)}}$$

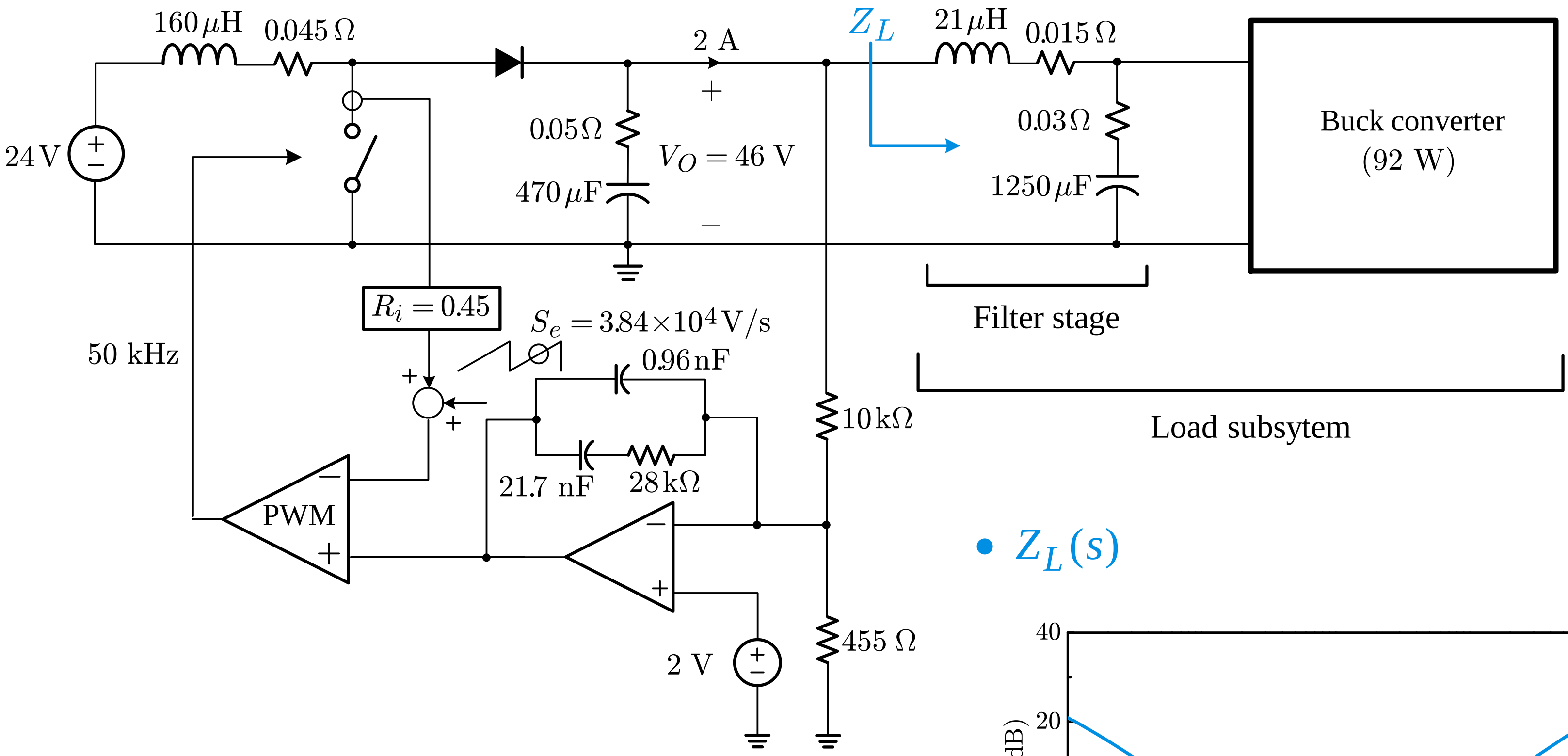
$$Z_{oL}(s) = Z_{oU}(s)\frac{1}{1+\dfrac{Z_{oU}(s)}{Z_L(s)}}$$

$$Z_{iL}(s) = Z_{iU}(s)\frac{1+\dfrac{Z_{oU}(s)}{Z_L(s)}}{1+\dfrac{Z'_{oU}(s)}{Z_L(s)}}$$

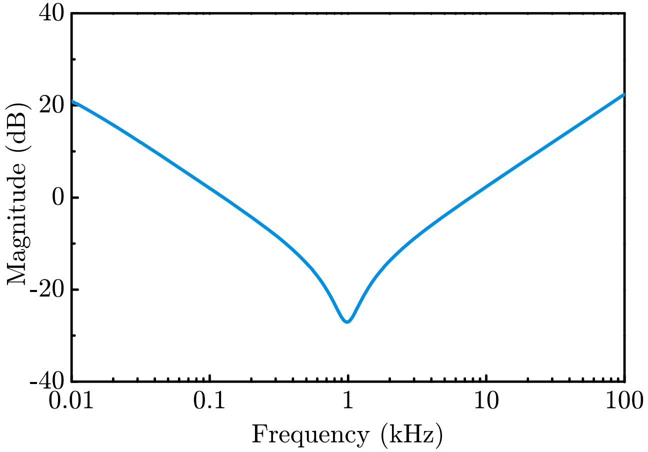


Z'_{oU}: closed-loop input-open output impedance

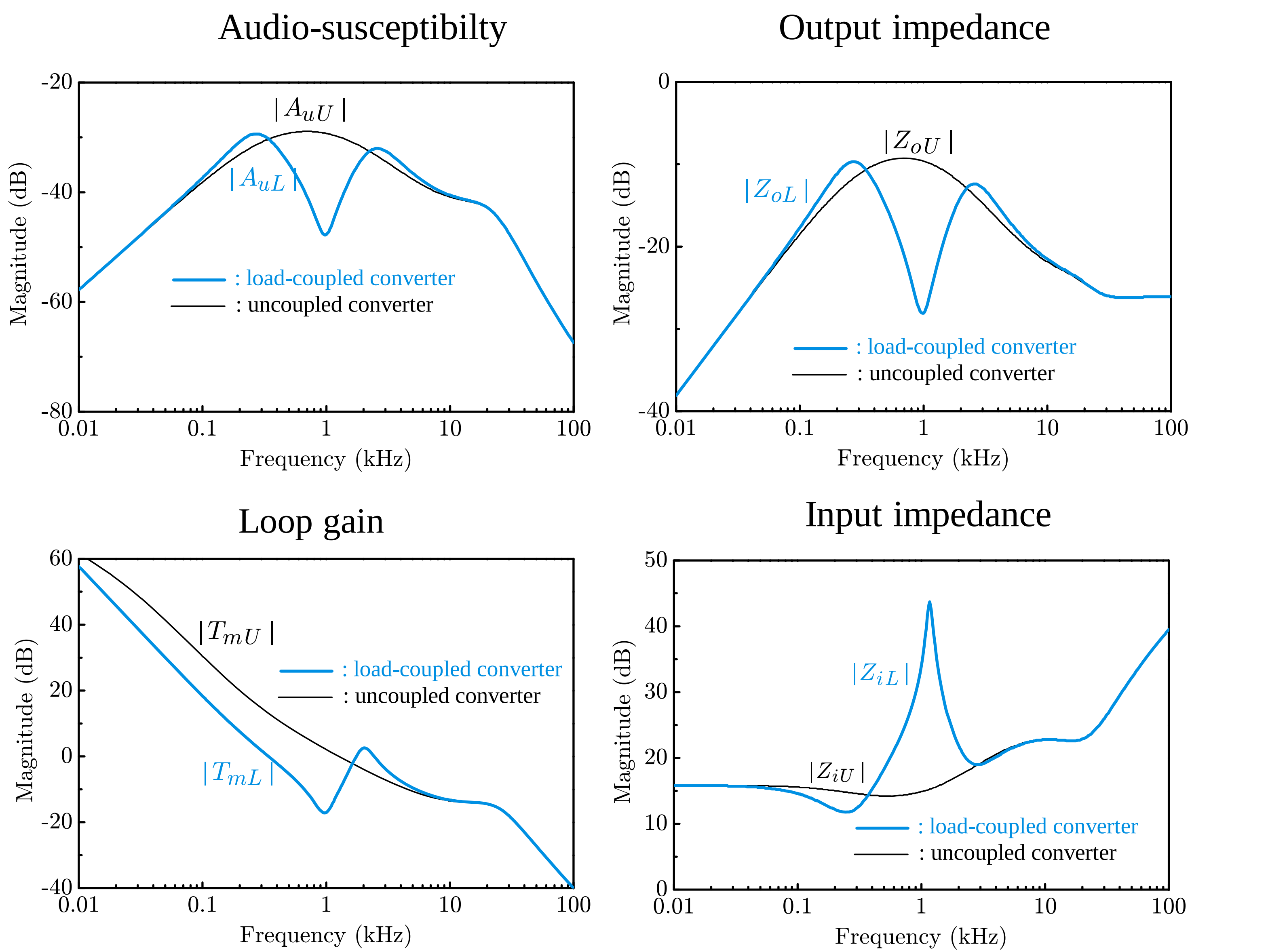
Performance of Load-Coupled Boost Converter



- $Z_L(s)$

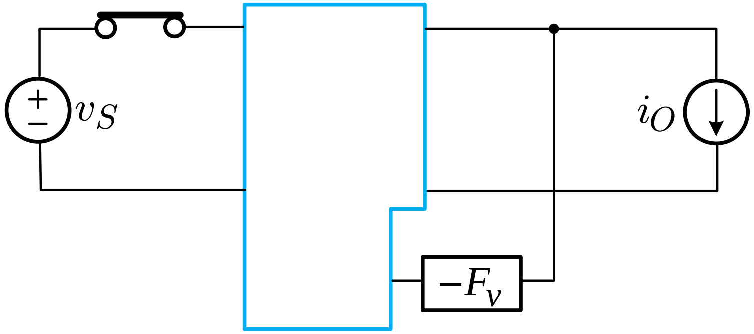


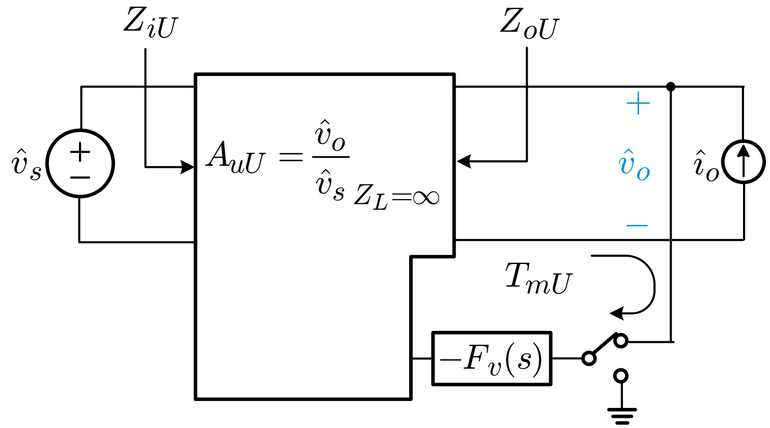
Performance of Load-Coupled Boost Converter



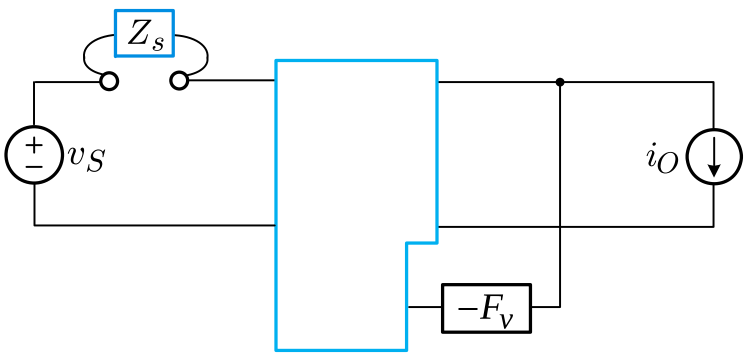
Source-Coupled Converter

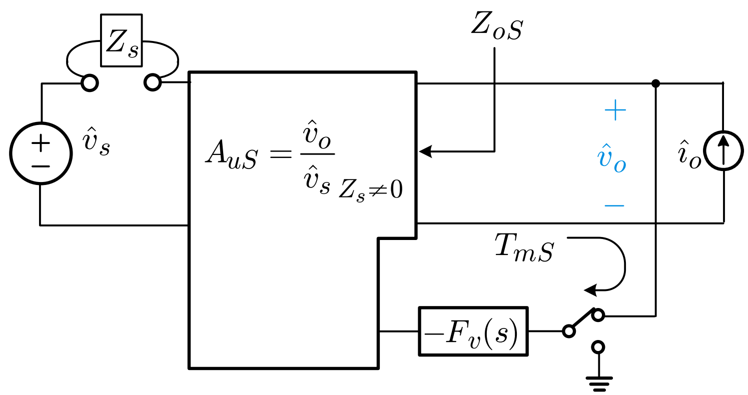
- Uncoupled converter



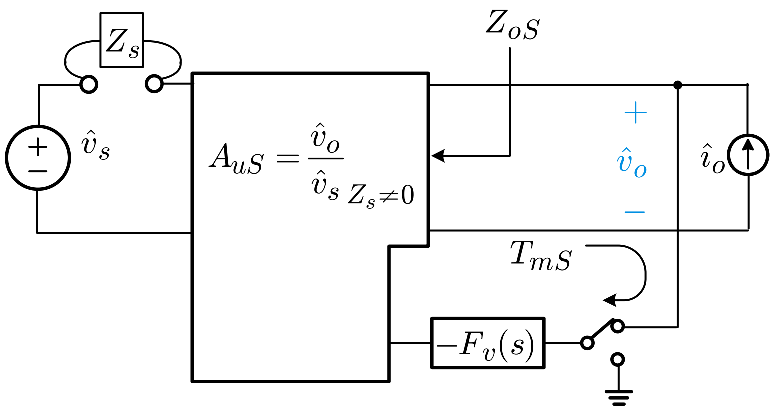


- Source-coupled converter





Performance of Source-Coupled Converter

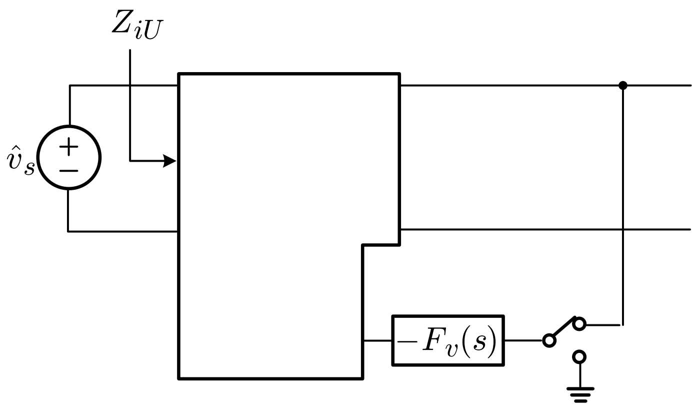


$$A_{uS}(s) = A_{uU}(s)\frac{1}{1+\dfrac{Z_s(s)}{Z_{iU}(s)}}$$

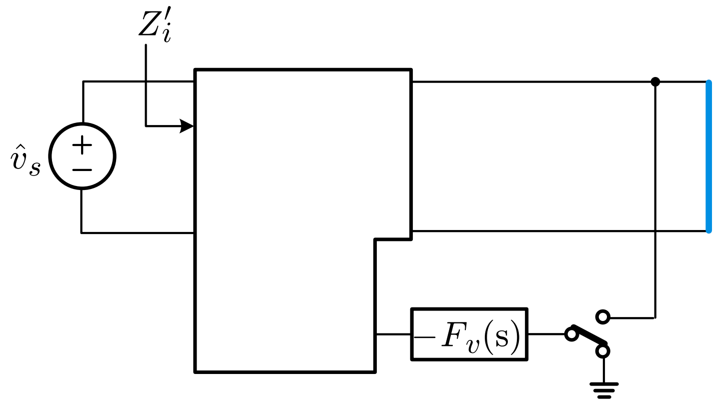
$$Z_{oS}(s) = Z_{oU}(s)\frac{1+\dfrac{Z_s(s)}{Z'_i(s)}}{1+\dfrac{Z_s(s)}{Z_{iU}(s)}}$$

$$T_{mS}(s) = T_{mU}(s)\frac{1+\dfrac{Z_s(s)}{Z''_i(s)}}{1+\dfrac{Z_s(s)}{Z'''_i(s)}}$$

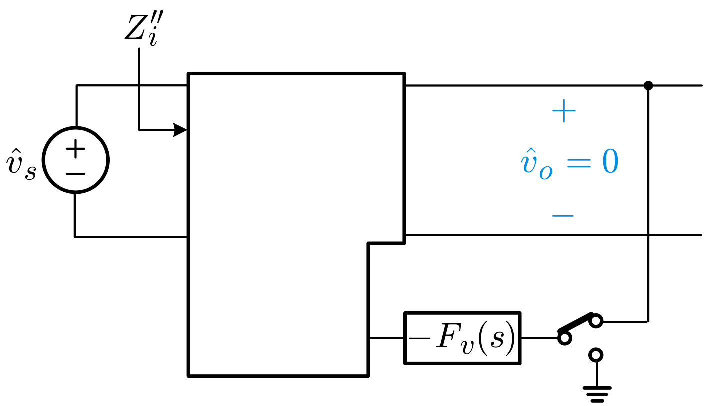
Input Impedance Definition



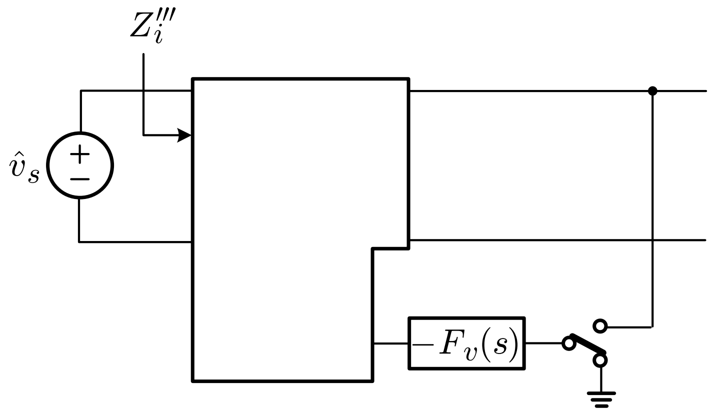
Z_{iU}: closed-loop input impedance



Z_i': open-loop output-shorted input impedance



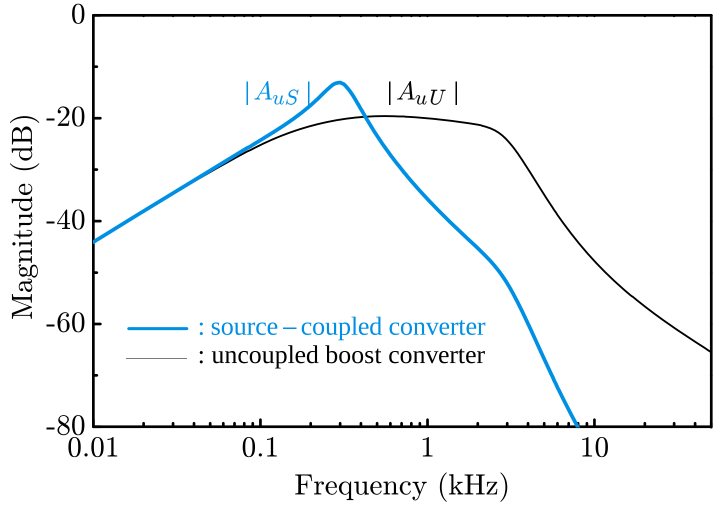
Z_i'': output-nullified input impedance



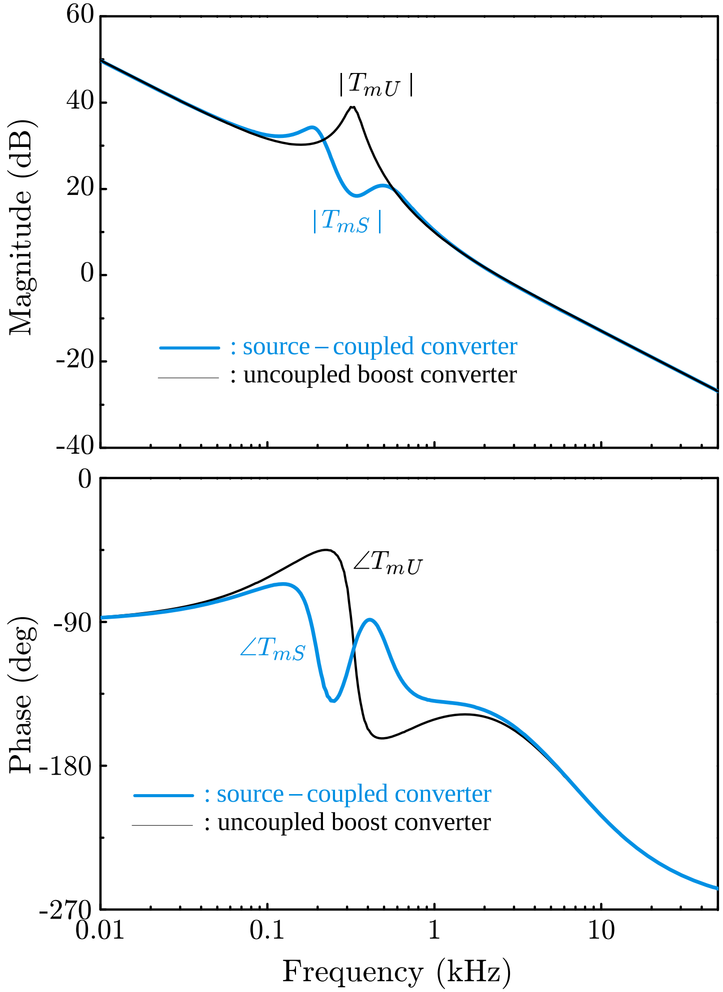
Z_i''': open-loop input impedance

Performance of Source-Coupled Boost Converter

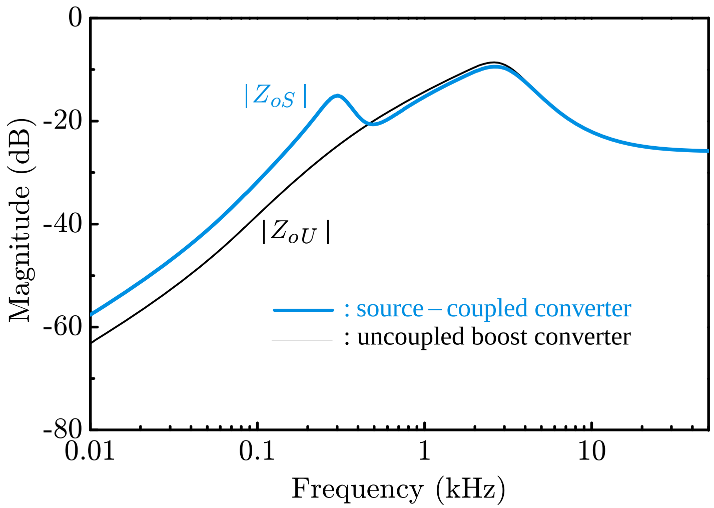
Audio-susceptibilty



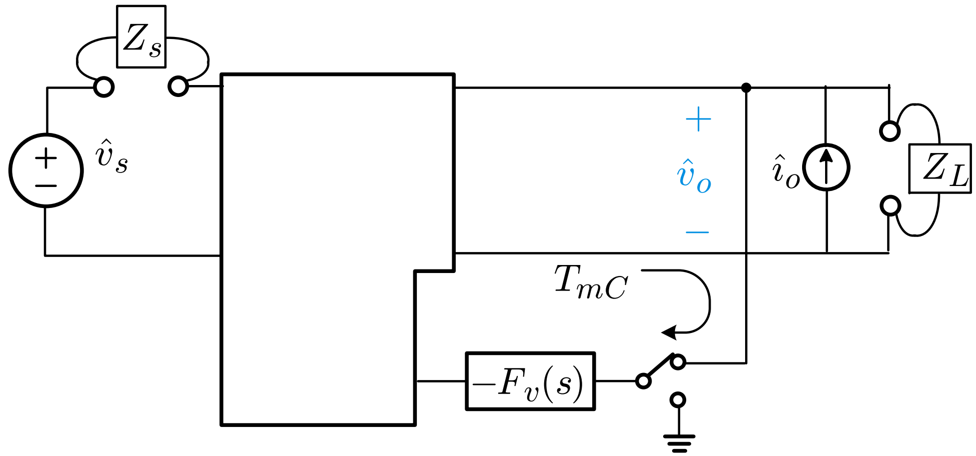
Loop gain



Output impedance



Source/Load-Coupled Converter

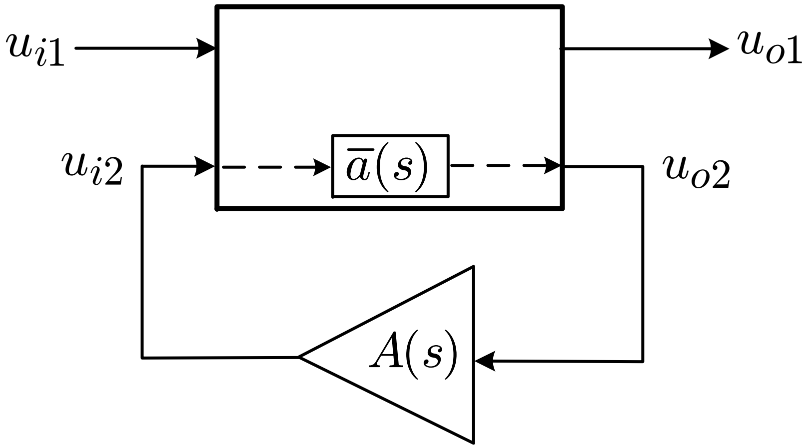


- Load-coupled converter: $T_{mL}(s) = T_{mU}(s)\dfrac{1}{1+\left(1+T_{mU}(s)\right)\dfrac{Z_{oU}(s)}{Z_L(s)}}$

- Source/load coupled converter:

$$T_{mC}(s) = \left(T_{mU}(s)\frac{1}{1+(1+T_{mU}(s))\dfrac{Z_{oU}(s)}{Z_L(s)}} \right)\left(\frac{1+\dfrac{Z_s(s)}{Z_i''(s)}}{1+\dfrac{Z_s(s)}{Z_i'''(s)}} \right)$$

Middlebrook's Feedback Theorem



- Two-output feedback controlled system

  $A(s)$: feedback gain

  $\overline{a}(s)$: forward gain

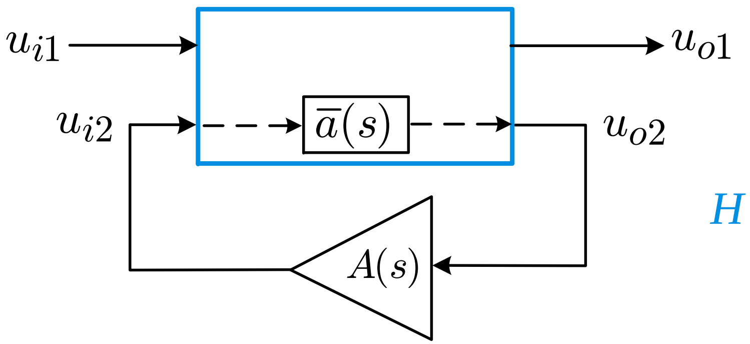
- $H(s) = \dfrac{u_{oi}(s)}{u_{i1}(s)} = H_{\infty}(s)\dfrac{T_m(s)}{1+T_m(s)} + H_0(s)\dfrac{1}{1+T_m(s)}$

  $H_{\infty}(s) = H(s)_{u_{o2}=0}$: feedback-signal nullified transfer gain

  $H_0(s) = H(s)_{A=0}$: open loop transfer gain

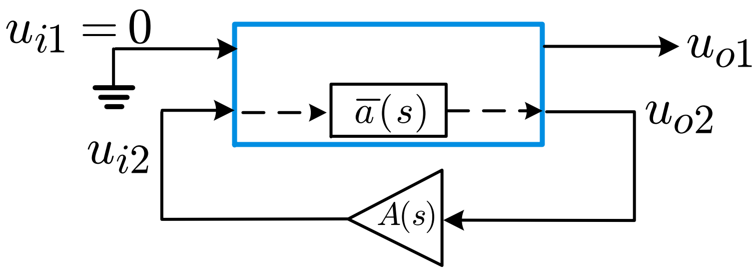
  $T_{mU}(s) = A(s)\overline{a}(s)$: loop gain

Pictorial Illumination



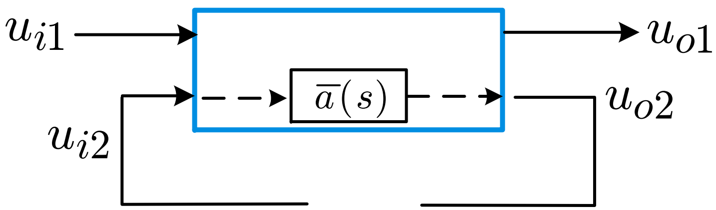
$$H(s) = H_\infty(s)\frac{T_m(s)}{1+T_m(s)} + H_0(s)\frac{1}{1+T_m(s)}$$

$T_m(s)$



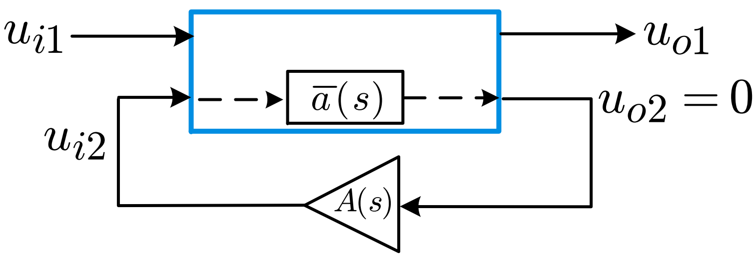
$$T_m(s) = A(s)\,\overline{a}(s)$$

$H_0(s)$



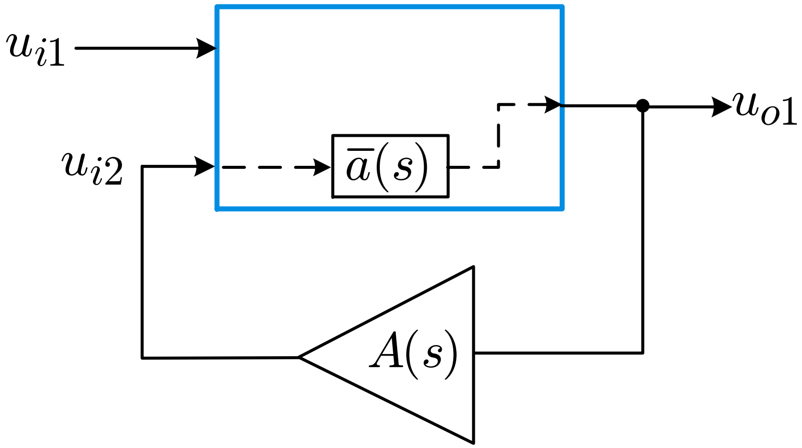
$$H_0(s) = \left.\frac{u_{o1}(s)}{u_{i1}(s)}\right|_{A(s)=0}$$

$H_\infty(s)$



$$H_\infty(s) = \left.\frac{u_{o1}(s)}{u_{i1}(s)}\right|_{u_{o2}=0}$$

Single-Output Feedback System

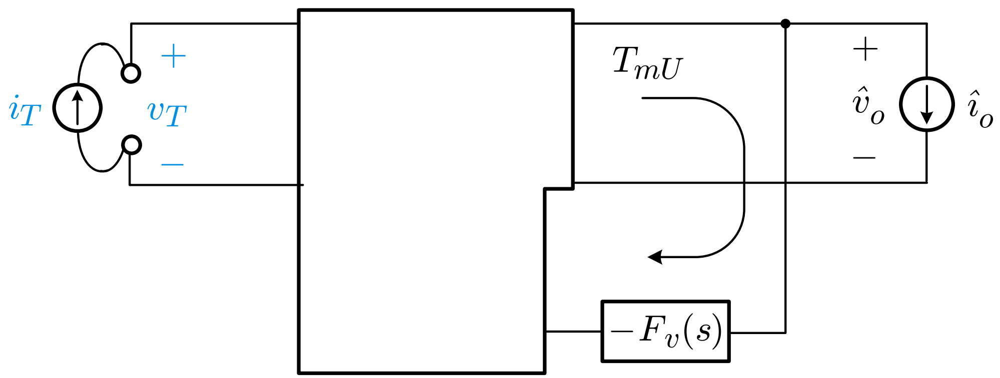


- $$H(s) = \frac{u_{o1}(s)}{u_{i1}(s)} = H_{\infty}(s)\frac{T_m(s)}{1+T_m(s)} + H_0(s)\frac{1}{1+T_m(s)}$$

$$H_{\infty}(s) = \left.\frac{u_{o1}(s)}{u_{i1}(s)}\right|_{u_{o1}(s)=0} = 0$$

- $$H(s) = \frac{u_{o1}(s)}{u_{i1}(s)} = H_0(s)\frac{1}{1+T_m(s)}$$

Input Impedance of Uncoupled Converter



- $$\frac{1}{Z_{iU}(s)} = \frac{i_T(s)}{v_T(s)} = \frac{1}{Z_i''(s)}\frac{T_{mU}(s)}{1+T_{mU}(s)} + \frac{1}{Z_i'''(s)}\frac{1}{1+T_{mU}(s)}$$

  $Z_i''(s)$: output nullified input impedance

  $Z_i'''(s)$: open-loop input impedance

  $T_{mU}(s)$: loop gain

- $$Z_{iU}(s) \approx \begin{cases} Z_i''(s) & \text{for frequencies where } |T_{mU}| \gg 1 \\ Z_i'''(s)G_{vs}(s) & \text{for frequencies where } |T_{mU}| \ll 1 \end{cases}$$

Chapter Summary

DC Power Distribution System

Uncoupled Converter

Power Stage Dynamics and Control Design of Uncoupled Converter

Coupled Converters and Middlebrook’s Extra Element Theorem

- Load-coupled converter
- Source-coupled converter
- Source/load-coupled converter

Middlebrook’s Feedback Theorem