

## 목 차

### 제 4 장. Further Array Topics

4-1. Scanning antennas.

4-2. Case study – Conformal Microstrip phased array for Aircraft test ;

- i ) Microstrip array design
- ii ) Phase shifter design

4-3. Frequency scanning antenna.

4-4. Orthogonal beams and multiple beam former (MBF).

4-5. Butler Matrix.

4-6. What is adaptive array?

## 제 4장. Further Array Topics

Advanced topics to be dealt with here.

- Scanning antennas.
  - i) phased array antenna.
  - ii) frequency scanning antenna.
- Orthogonal beams and multiple beam formers.
- What is adaptive array?

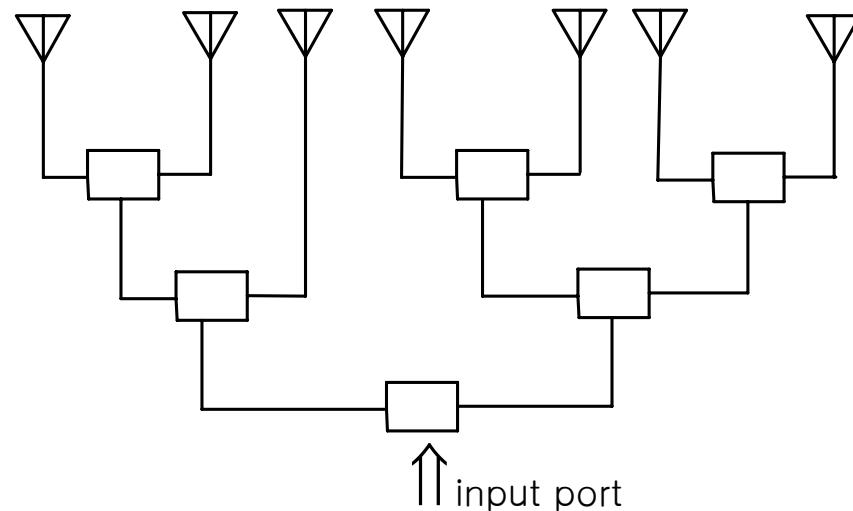
### 4-1. Scanning antennas

#### i) Phased array antennas

Large antenna systems are difficult to mechanically scan in a rapid fashion, and because of this, electronically scanned arrays have been developed.

These arrays, which may have several thousand elements, are scanned by incorporating either ferrite or diode phase shifter in each feed line.

- Standard configuration for a linear array with a “corporate” or “parallel” amplitude distribution network, and a phase shifter for each array element.



corporate (parallel) feed network.

Key ingredients :

- i) power divider circuit.
- ii) phase shifter to vary the array phase distribution, for example, to steer the beam electronically.

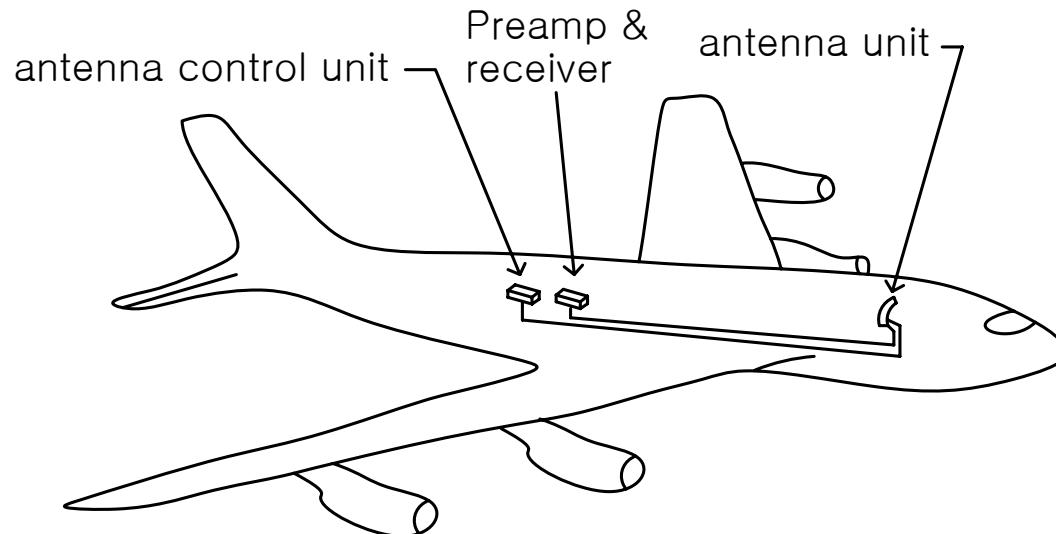
## 4-2. Case study-Conformal Microstrip phased array for Aircraft Test with ATS-6.

(ref; G.G. Sanford, "Conformal microstrip phased array for aircraft tests with ATS-6". IEEE Trans. AP. vol. 26, no. 5, pp.642-646, Sept. 1978)

; One of the primary experiments planned for the ATS-6 satellite was an aircraft location and communication experiment.

Design goal :

A single phased array mounted on one side of the aircraft satisfying the required specifications.



Requirements for the array;

- operation from 1543.5 to 1558.5 MHz ;
- VSWR < 2.1 referenced to  $50\Omega$  ;
- fan-shaped beam, narrow in elevation plane and wide in azimuth plane;
- 12 dBi right-hand circular gain;
- right-hand circular polarization with 3 dB maximum axial ratio;
- sidelobes minimized in ocean multipath direction;
- 3 bit digital phase shifter resolution ; and
- aircraft operation at temperatures between +50°C and –50°C, altitude to 50000 feet, humidity to 100 percent relative, and vibration levels characteristic of the mounting area.

In order to make the array practical for commercial application, it must conform to four qualitative requirement

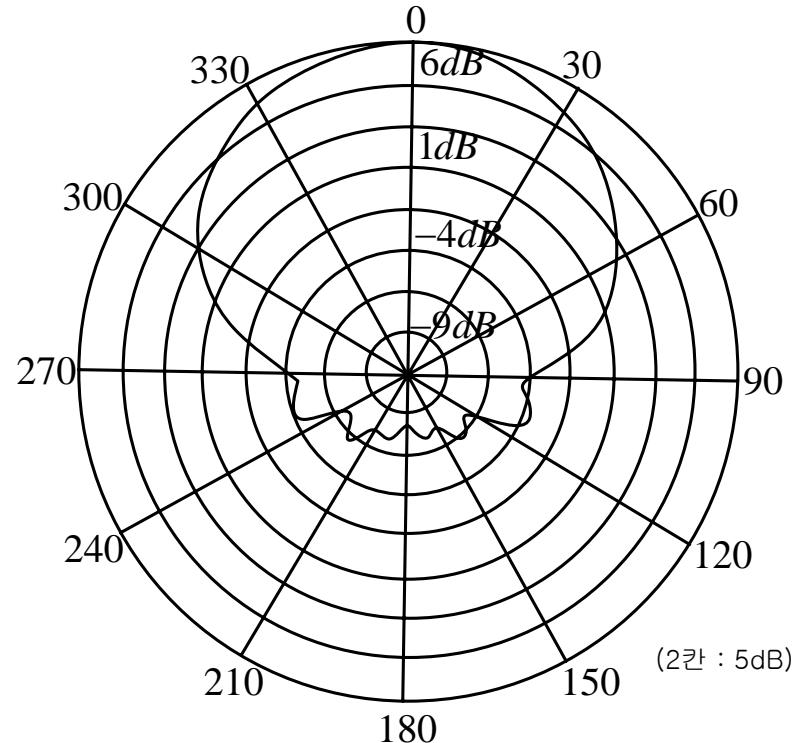
- producible in quantity at low cost;
- void of any significant effect on the aircraft structural or aerodynamic characteristics;
- easy to install and replace on the aircraft ; and
- inexpensive to operate and maintain.

① Selection of the radiation element.

- conformal.

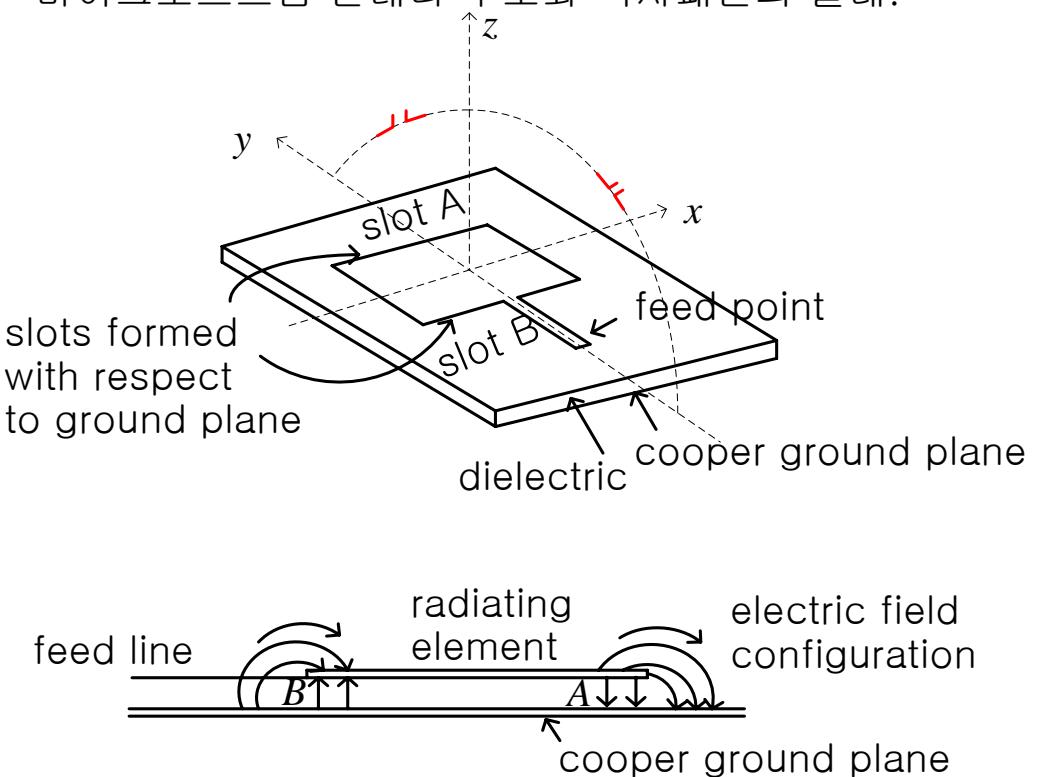
∴ Most suitable type.

→ microstrip antenna.

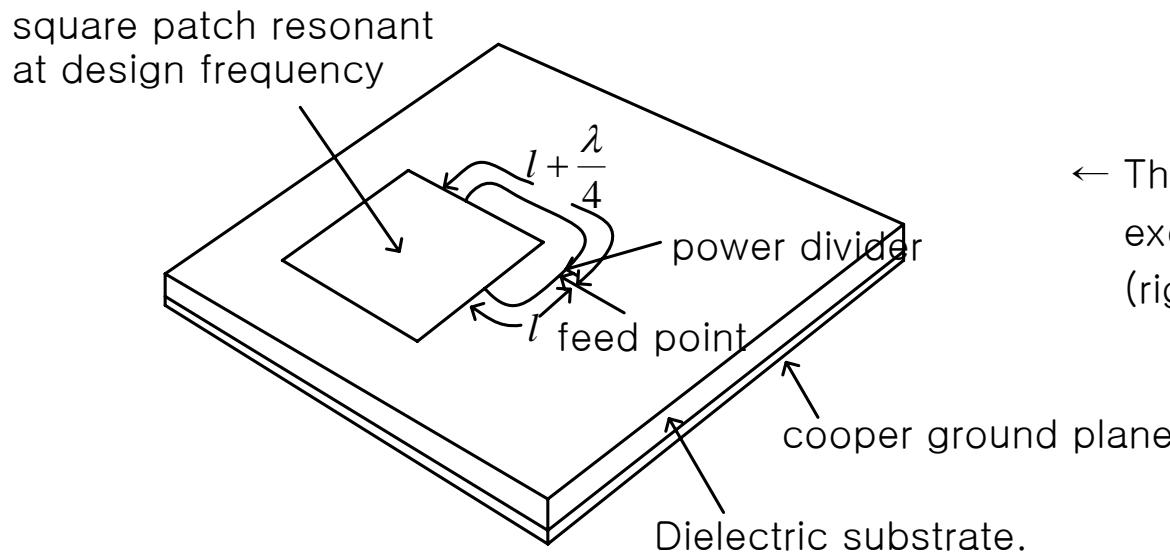


단일 patch의 E-plane radiation pattern. (y-z plane)

마이크로스트립 안테나 구조와 복사패턴의 일례.



Two slot model of the rectangular microstrip patch antenna chosen as a radiating element.



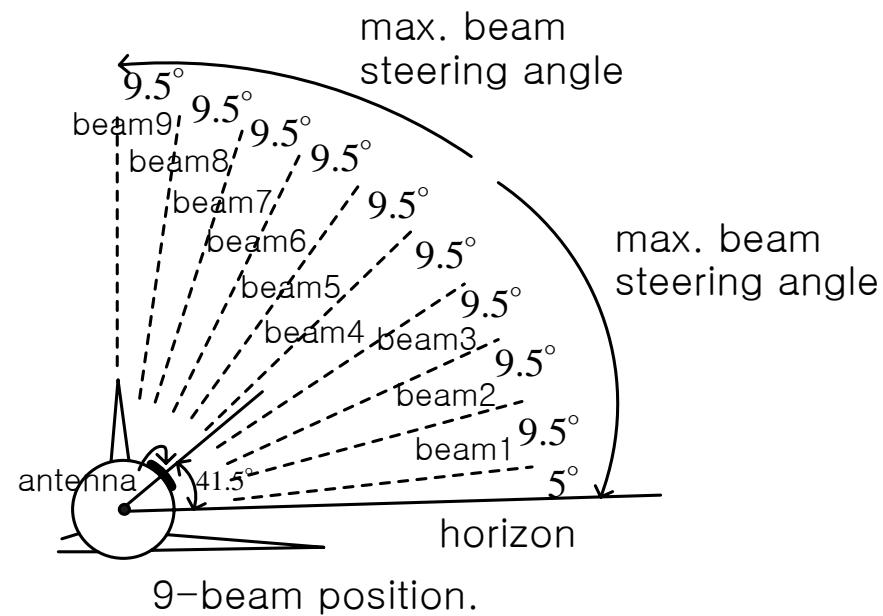
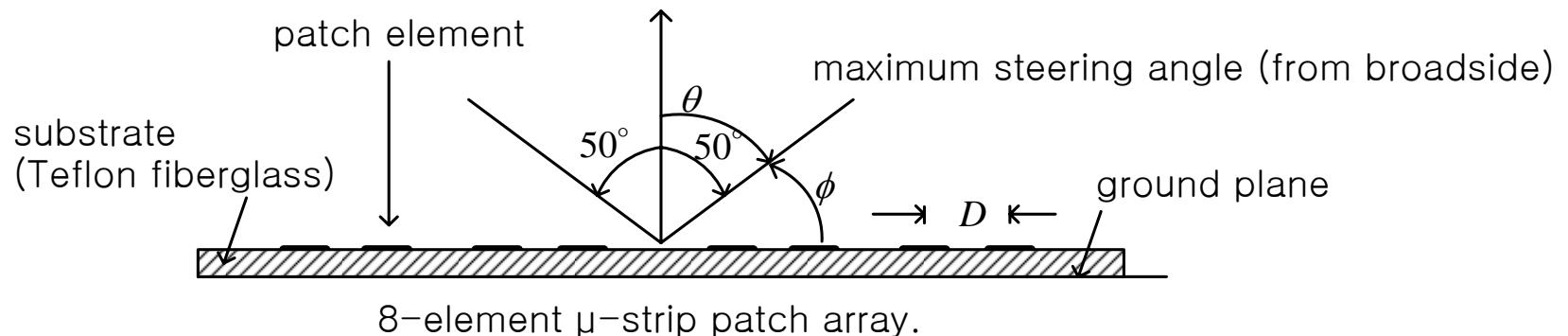
← The two orthogonal modes must be excited equally, in phase quadrature.  
(right handed circular polarization)

Circularly polarized microstrip patch antenna.

## ② Array design

- Suppression of grating lobe

$$D \leq \frac{\lambda}{(1 + \sin \theta)} \quad \text{여기에서 } D = \text{separation} \quad \lambda = \text{wavelength} \quad \theta = \text{max. beam steering angle}$$



주의)  $D \leq \frac{\lambda}{(1 + \cos \phi)}$  ; 2장에서는  $\phi$ 로 각도 표시

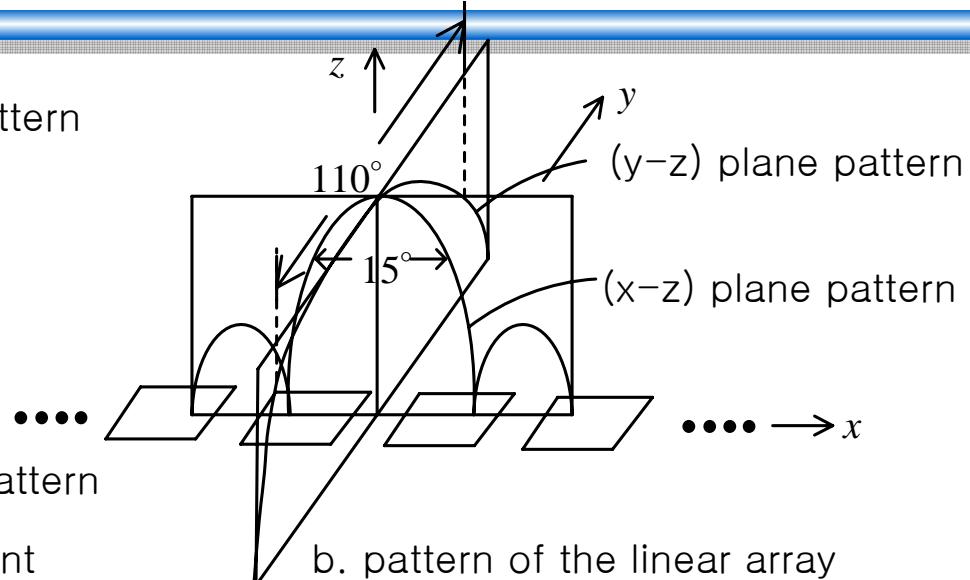
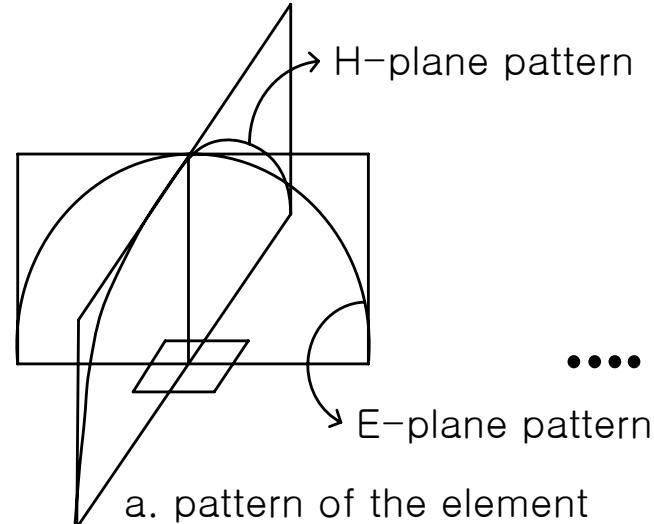
maximum beam steering angle  $\theta = 50^\circ$  이므로

$$D \leq \frac{\lambda}{(1 + \sin 50^\circ)} = 0.5662\lambda$$

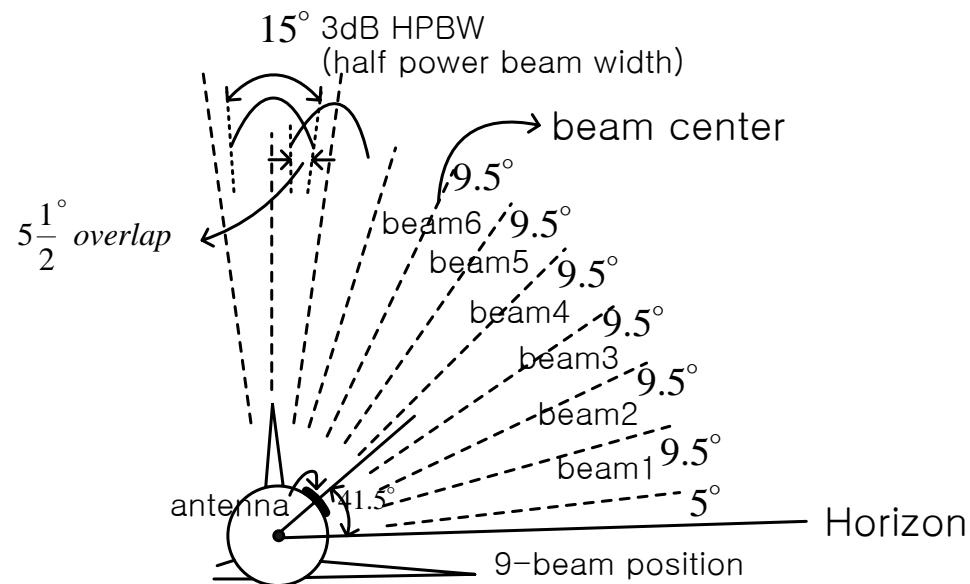
$\therefore D = 0.52\lambda$  로 선택.

1 x 8 linear array로 선택하고 radiation pattern이

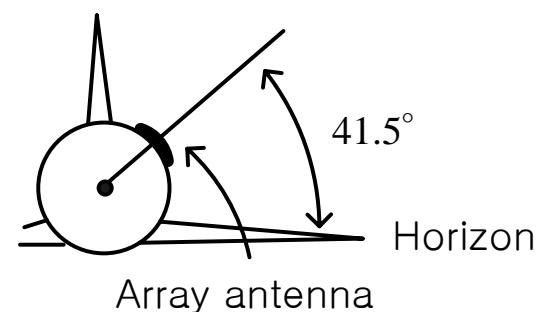
$15^\circ \times 110^\circ$  beam이 되도록 설계.(아래 그림 참조)



principle plane (E-and H-plane) pattern을  $15^\circ \times 110^\circ$ beam01 되도록 선택.  
[3dB HPBW]



그림에서와 같이,  
beam width가  $15^\circ$ 인 beam01  
 $5\frac{1}{2}$  overlap 되어 있다.



③ phase shifter design

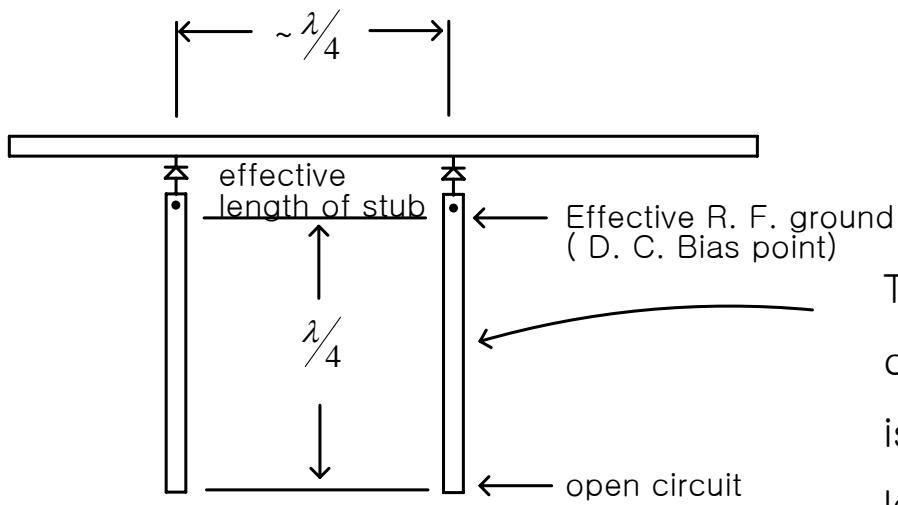
- Constant phase progression with  $9\frac{1}{2}^\circ$  steering increments and  $0.52\lambda$  spacing between elements.
- With eight-element array, it is possible to select three bits for each element that will provide the desired phase progression.
- The phase shifter for each element has three of the following bits :  $30^\circ$ ,  $60^\circ$ ,  $120^\circ$ , and  $180^\circ$ .
- The arrangement of these bits and the resulting phase progression for each beam position are shown in the table below.

Effective 30° phase resolution using 3-bit phase shifters

BEAM POSITIONS													
Element Number	Phase	Bits	Required		1	2	3	4	5	6	7	8	9
1	60	120	180		120	120	240	0	0	240	0	120	0
2	(30	60	120)	+180*	240	210	300	30	0	210	300	30	240
3	60	120	180		0	300	0	60	0	180	240	300	120
4	30	60	120		120	30	60	90	0	150	180	210	0
5	60	120	180		240	120	120	120	0	120	120	120	240
6	30	60	120		0	210	180	150	0	90	60	30	120
7	60	120	180		120	300	240	180	0	60	0	300	0
8	(30	60	120)	+180*	240	30	300	210	0	30	300	210	240

\* Fixed 180° phased shifter

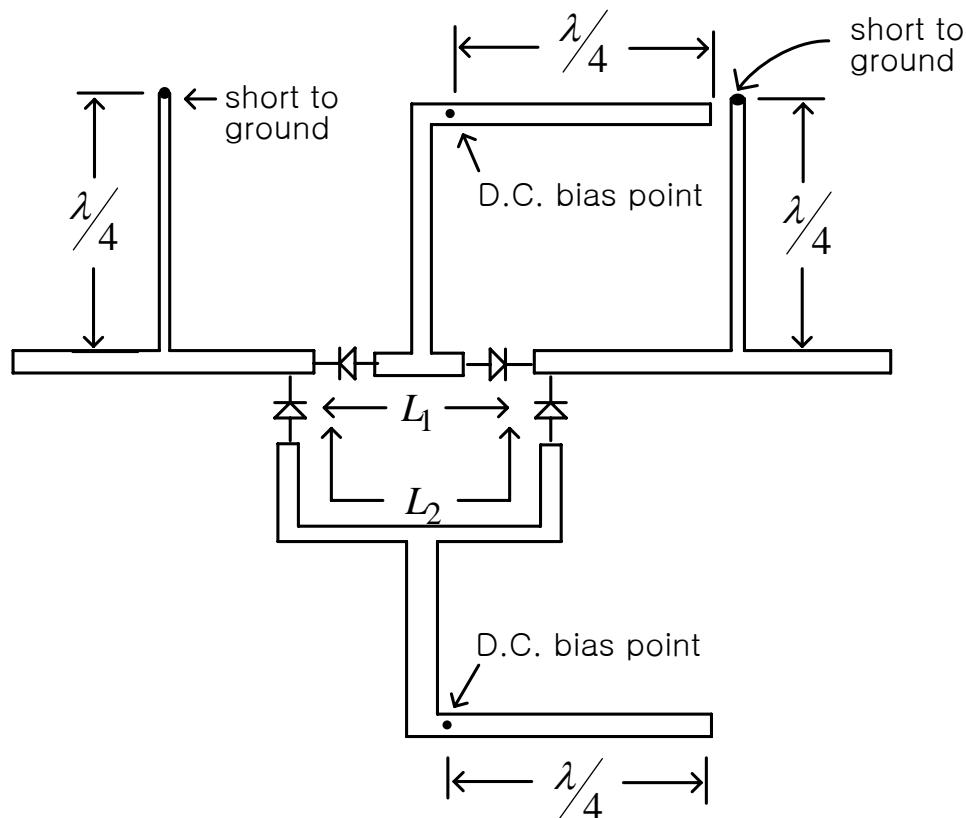
- phase shifter circuit selection
  - i)  $30^\circ$  bit  $\sim$  a loaded line phase shifter



The additional phase shift due to the capacitance of the back biased diodes is compensated by adjusting the stub lengths empirically.

disadvantage) large  $I^2R$  loss for the diodes in the forward bias state of the pin diode.

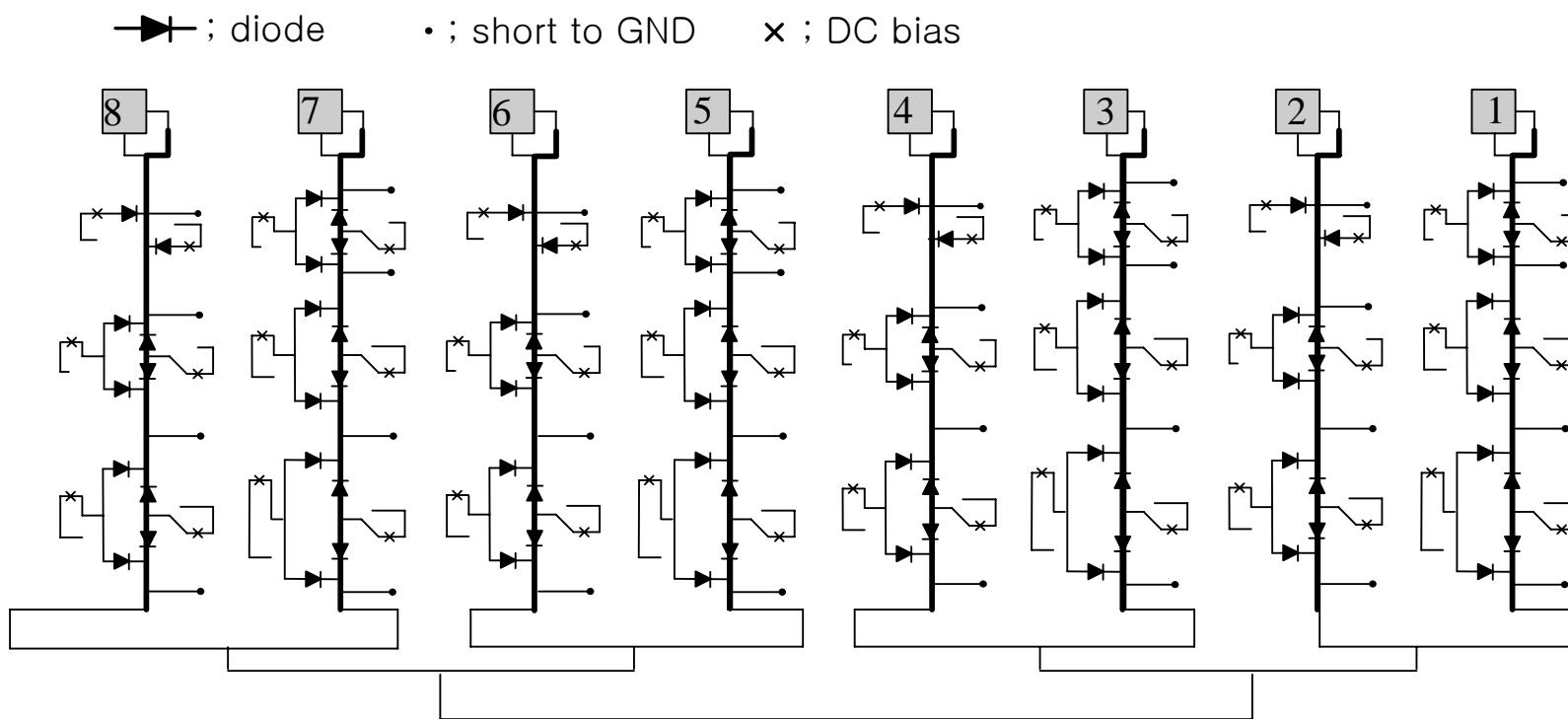
ii)  $60^\circ$ ,  $120^\circ$ , and  $180^\circ$  bits ~ switched line phase shifters



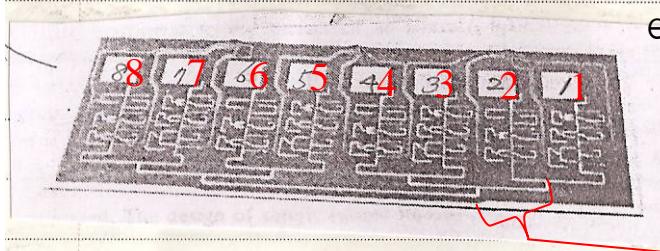
- The phase shift  $\propto$  difference between  $L_1$  and  $L_2$ .
- $L_1$  should be as short as possible for  $60^\circ$  and  $120^\circ$  bits.  
(Length  $L_1$  for the  $180^\circ$  bit is less critical)

Note) phase shifter circuit – 100 Ohm characteristic impedance

- i ) reduction of insertion loss (다이오드 저항 대 선로 특성 임피던스 비 낮춤).
- ii ) power divider network design 용이.
- iii ) 선로 폭을 reasonable하게 유지할 수 있다는 점.



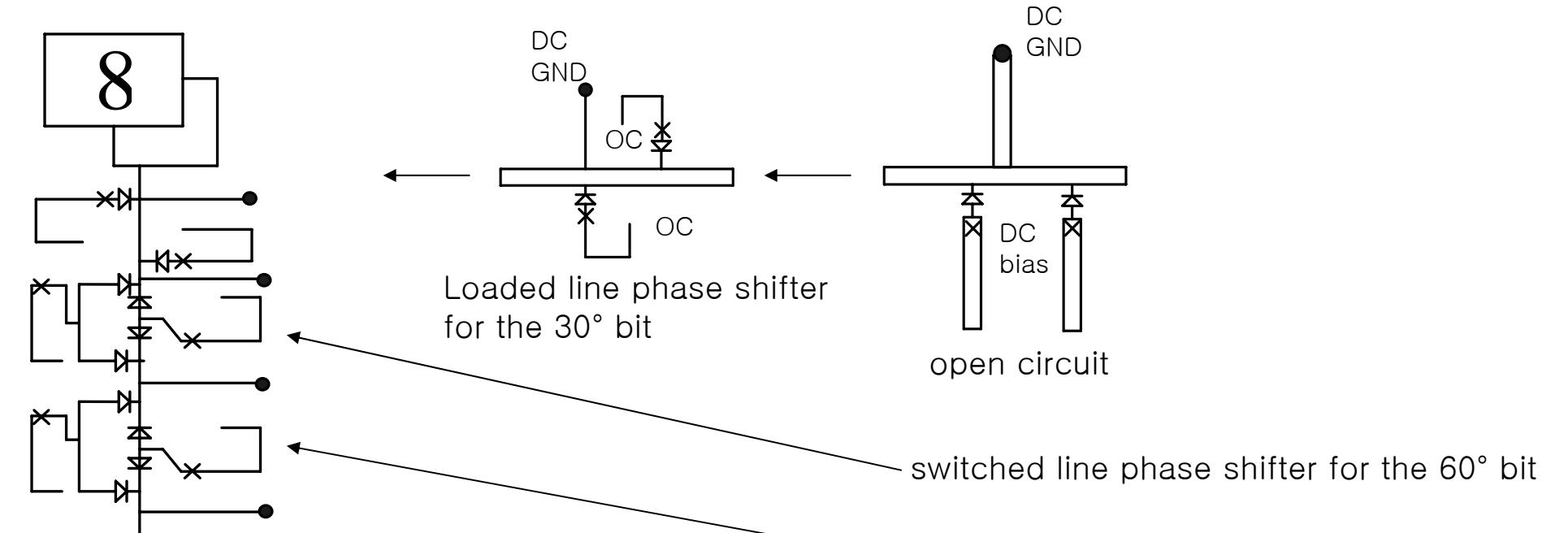
photograph of the completed array along side an unfinished antenna board showing the entire circuit.



- gain 12dBi
- polarization axial ratio < 2dB for most beam positions and frequencies.
- VSWR < 1.8

power dividing circuit with desired fixed phase progression.

④ Detailed view on the 8th element



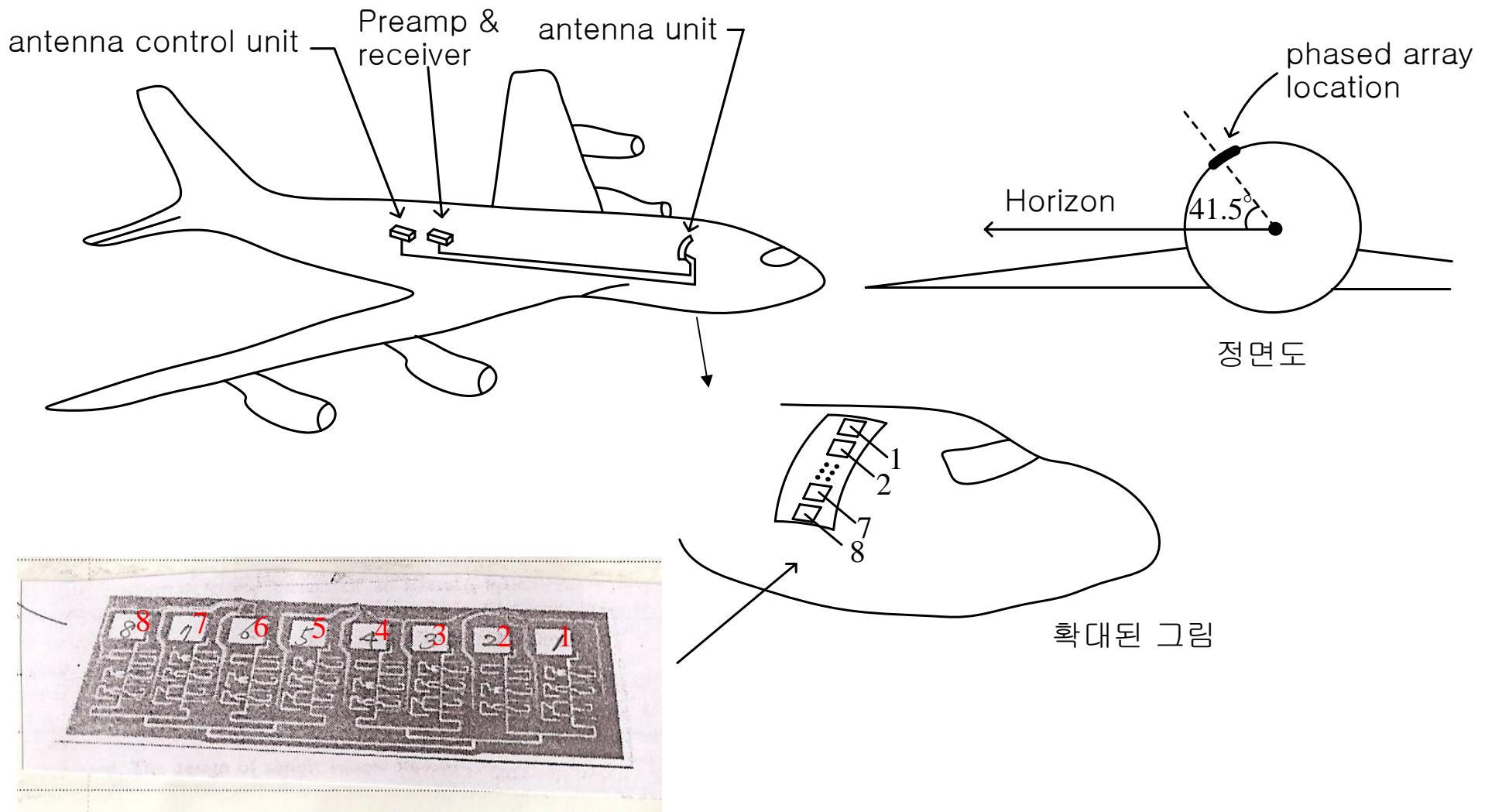
X ; DC bias.

● ; short to GND.

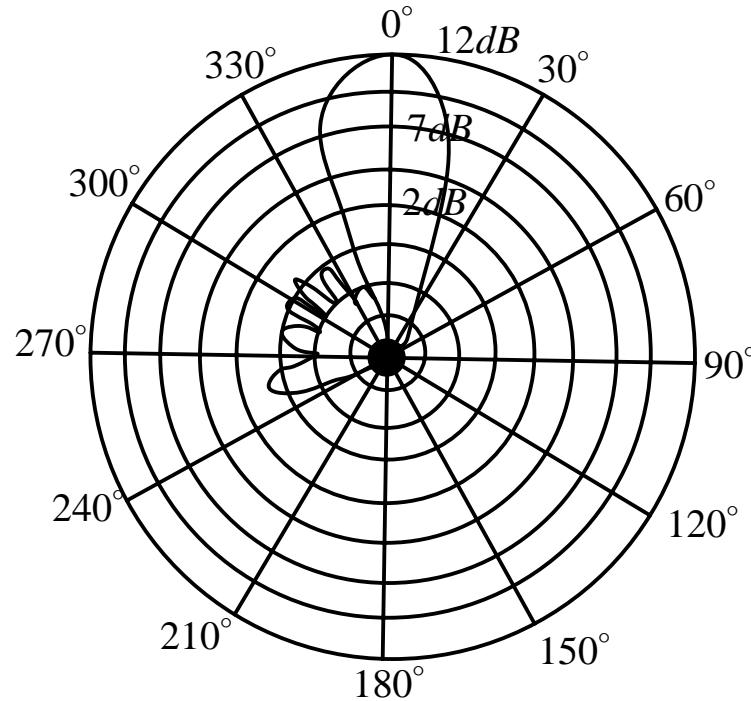
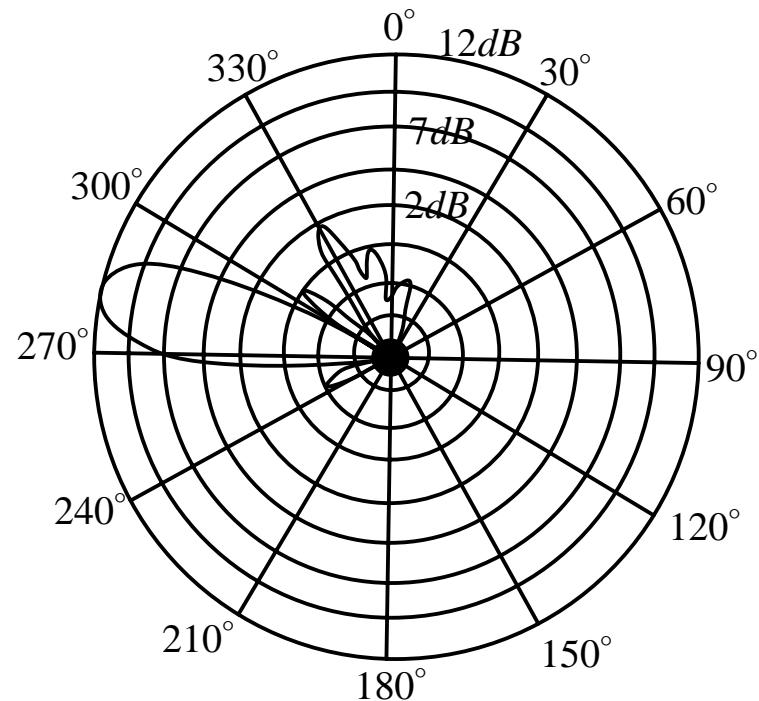
note) - 사용공간을 줄이기 위하여 open stub의 뒷부분의 bending구조 채택.

- $60^\circ$ bit와  $120^\circ$ bit 사이의 ground는 공통으로 구성.
- pin diode 2개로 구성되어 있는 bit는  $30^\circ$  phase shifter임.

실제장착구조 :



Radiation pattern (electronically steerable in elevation).

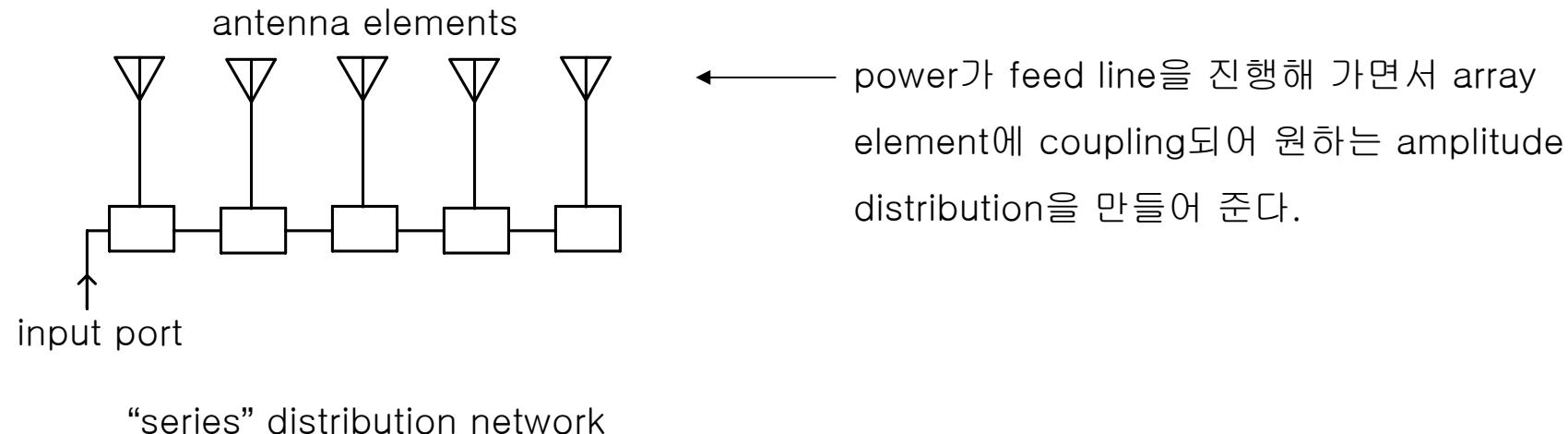


measured radiation patterns in elevation for two representative beam positions.

Antenna test

; performance in terms of bit-error rate(BER), multipath rejection, ranging, and voice intelligibility.

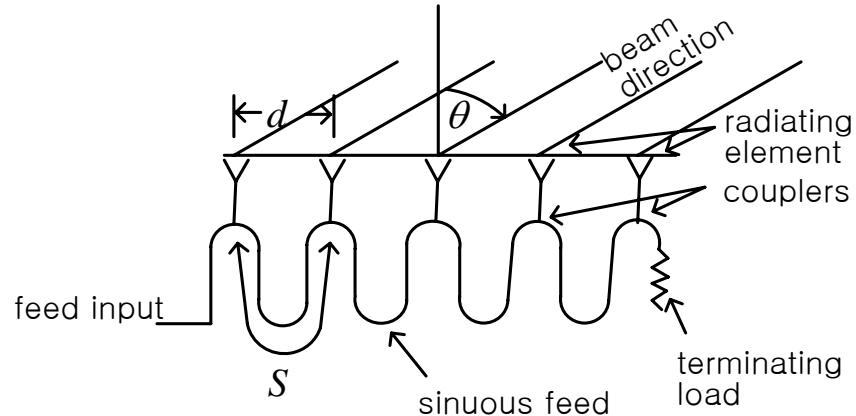
#### 4-3. frequency scanning antenna.



FS(frequency scanning) antenna

→주파수에 따라 main beam 방향이 변화되는 antenna.

이웃한 소자간 r.f path length는 어떤 특정주파수  $f_0$ 에서 1 파장  $\lambda_0$  라고 하면, 주파수  $f_0$ 에서 각 소자들은 동상(co-phased)이 되어 broadside beam을 복사.



General schematic diagram of a series fed frequency-scan antenna. 여기에서  $f = f_0$  이면  $s = \lambda_0$  가 되어 broadside beam 형성.

소자간 간격이  $d$  이고 위상차가  $\phi (> 0)$  일 때, 그림에서와 같이 broadside 방향에서  $\theta$  만큼 편향 (deflected) 되었다고 하면



Note)  $\phi > 0$  이려면  $\lambda < \lambda_0$ , 즉  $f > f_0$  가 되고 그림에서  $\theta < 0$  가 된다.

$$\sin \theta = \frac{-(\phi/d)}{k_0} = \frac{-(\phi/d)}{2\pi/\lambda} = -\frac{\phi\lambda}{2\pi d} = \frac{-2\pi\left(\frac{\lambda_0}{\lambda}-1\right)\lambda}{2\pi d} = \frac{(\lambda-\lambda_0)}{d} .$$

파장이  $\lambda$ 이고 주파수가  $f (\neq f_0)$ 인 경우,  
이웃한 소자간의 phase difference  $\phi$  는

$$\phi = 2\pi\left(\frac{\lambda_0}{\lambda}-1\right) .$$

$$\therefore \sin \theta = \frac{\lambda - \lambda_0}{d} \text{ 로 부터}$$

파장(또는 주파수)이 변함에 따라 maximum beam direction이 변화함을 볼 수 있다.

이러한 효과는 활용분야에 따라 장단점이 될수 있다.

먼저 frequency scanning antenna로 사용할 경우, phase shifter도 필요없고 급전방식도 직렬로서 간단하며 회로공간도 대폭축소 된다.

Space for circuitry

반면에 전파의 복사 방향이 고정되어 사용되는 경우, 중심주파수 ( $f_0$ )에서 벗어나면 요구되는 복사 방향에서 벗어난다. 요구되는 복사방향으로부터 반치각(HPBW) 방향으로 향하게 되는 경우, 신호출력 레벨도 3dB 떨어지므로, system bandwidth에 중대한 제약이 된다.

ex) 매우 좁은 빔폭이 요구되는 array antenna가 있다고 하고,  $f = f_0 + \frac{\Delta f}{2}$ 에 beam01 broadside 방향에서  $\alpha$ 의 각도만큼 편향(deflection)된다고 하면

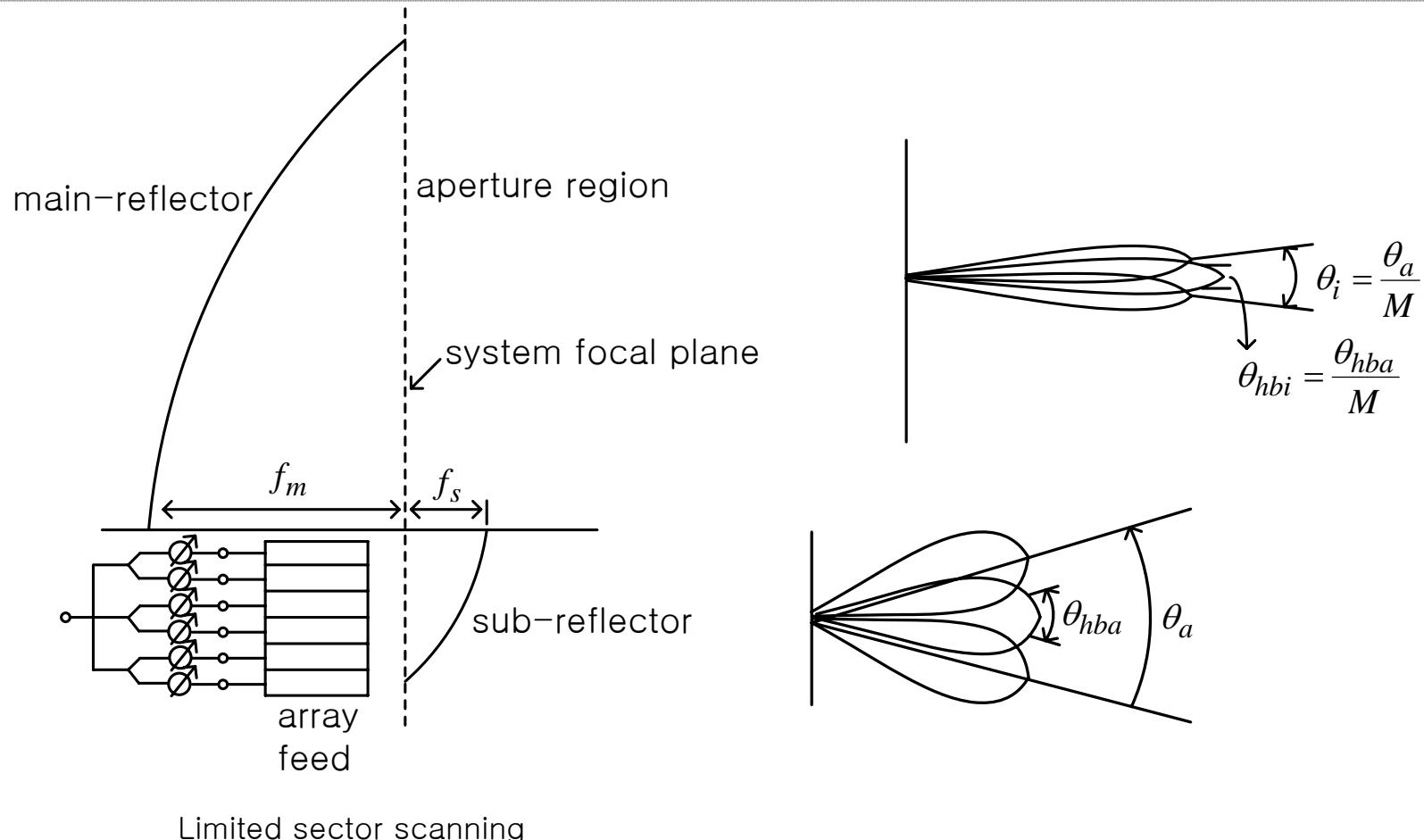
$$\begin{aligned}
 |\sin \alpha| &\approx |\alpha| \approx \left| \frac{(\lambda - \lambda_0)}{d} \right| = \left| \left( \frac{\lambda}{\lambda_0} - 1 \right) \right| \cdot \frac{\lambda_0}{d} \\
 &= \left| \left( \frac{f_0}{f} - 1 \right) \right| \cdot \frac{\lambda_0}{d} = \frac{|f_0 - f|}{f} \cdot \frac{\lambda_0}{d} \\
 &= \frac{|f - f_0|}{f_0} \cdot \frac{\lambda_0}{d} (\because f_0 \text{ 근처에서}, f = f_0) \\
 &= \frac{\Delta f}{f_0} \cdot \frac{\lambda_0}{2d} \left( \because |f - f_0| \approx \frac{\Delta f}{2} \right).
 \end{aligned}$$

만일  $d = \frac{\lambda_0}{2}$  이고 beamwidth가  $2^\circ$ 라고 하면,  $\alpha = \pm 1^\circ$  일 때, 신호출력은 3dB 감소된다.

이 경우  $|\alpha| = |\pm 1^\circ| \approx \frac{\Delta f}{f_0} \cdot \underbrace{\frac{\lambda_0}{2d}}_{=1} = \frac{\pi}{180}$

$$\therefore \frac{\Delta f}{f_0} \approx \frac{\pi}{180} \approx 1.8\% .$$

- 빔폭이 매우 좁고, 고이득(high gain)이 요구되는 경우, 매우 큰 phased array가 요구될 수 있다. 고 이득의 빔은 reflector antenna를 사용하면 용이하게 얻을 수 있는데 이렇게 되면 mechanical scanning을 사용해야 한다. 이런 경우, 비교적 넓은 범위의 scan angle을 지닌 소형 phased array와 대형 reflector 안테나를 결합하여 high gain을 유지하면서 제한된 각도 범위에서의 scanning(limited sector scanning)이 가능한 compromise arrangement를 사용할 수 있다.

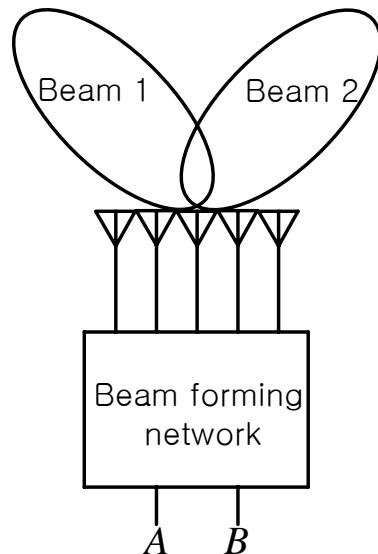


Note) feeding array 만으로 복사할 때의 aperture에 비하여 imaging arrangement (sub-reflector + main reflector)를 사용하는 경우의 effective aperture 가 훨씬 크다.

→ 만일 M배로 커진다면 beam width 도  $\frac{1}{M}$  로 줄고 scan angle도  $\frac{1}{M}$  로 줄게된다.

#### 4-4. Orthogonal beam and multiple beam formers(MBF)

아래 그림은 multiple beam former(MBF)와 관련되어 사용되는 linear array를 나타낸다.

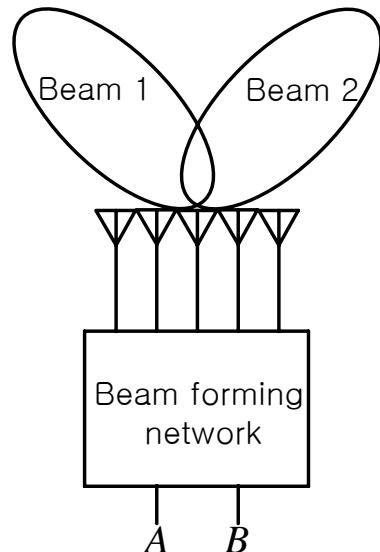


multiple beam former(MBF)

주어진 주파수에 대하여, 고정된 방향으로 형성된 beam들이 동시에 있기 때문에 각 beam port에 receiver가 연결되어 있으면 동시에 여러 방향으로 부터 오는 신호들을 수신할 수 있다.

MBF의 구성 :

- power divider/combiners at each array element.
  - separate networks for each independent beam.
- multiple beams이 서로 orthogonal 하면 simultaneous MBF 구성 가능.
  - zero mutual coupling between beam ports.



~ port A에 단위 전압 인가시  
; 전계 pattern  $E_A(\theta)$ .

~ port B에 단위 전압 인가시  
; 전계 pattern  $E_B(\theta)$ .

- port A, B를 동시에 구동한 경우의 전계의 radiation pattern,  $E_A(\theta) + E_B(\theta)$ ;

A, B port간에 coupling이 없는 경우,

$$\begin{aligned}
 & \int [E_A(\theta) + E_B(\theta)] \cdot [E_A(\theta) + E_B(\theta)]^* d\theta \\
 &= \int [E_A(\theta) + E_B(\theta)] \cdot [E_A^*(\theta) + E_B^*(\theta)] d\theta \\
 &= \int E_A(\theta) \cdot E_A^*(\theta) d\theta + \int_s E_B(\theta) \cdot E_B^*(\theta) d\theta + \int E_A(\theta) \cdot E_B^*(\theta) d\theta + \underbrace{\int E_A^*(\theta) \cdot E_B(\theta) d\theta}_{\left\{ \int E_A(\theta) \cdot E_B^*(\theta) d\theta \right\}^*} .
 \end{aligned}$$

여기에서 beam이 서로 orthogonal 하기 위한 조건은, 즉  $\int E_A(\theta) \cdot E_B^*(\theta) d\theta + \left\{ \int E_A(\theta) \cdot E_B^*(\theta) d\theta \right\}^* = 0$  이 되려면

$$\int E_A(\theta) \cdot E_B^*(\theta) d\theta = 0 .$$

ex) aperture의 길이가  $L$ 이고 aperture distribution이  $f(x)$ 인 경우,

전계의 radiation pattern :

$$E(S = \sin \theta) = \int_{-L/2}^{L/2} f(x) \exp(jkxS) dx$$

beam의 peak가 각각  $S = -\sin \alpha$ ,  $s = \sin \alpha$ 인 pattern을 주는 두 aperture distribution은

$$g_A(x) = f(x) \exp(jk x \sin \alpha)$$

$$g_B(x) = f(x) \exp(-jk x \sin \alpha).$$

$g_A(x)$  와  $g_B(x)$ 에 의한 두 beam이 orthogonal 하려면

$$\int E_A(S) \cdot E_B^*(S) dS . \quad \dots \dots \dots \quad (\text{A})$$

$$\left. \begin{aligned} \text{여기에서 } E_A(S) &= \int_{-L/2}^{L/2} f(x) \exp\{jkx(S + \sin \alpha)\} dx \\ E_B(S') &= \int_{-L/2}^{L/2} f(y) \exp\{jkx(S' - \sin \alpha)\} dy \end{aligned} \right\} \dots \dots \dots \quad (\text{B})$$

(B)→(A) 대입;

$$\begin{aligned}
 & \int \left[ \underbrace{\int_{-L/2}^{L/2} f(x) \exp\{jkx(S + \sin \alpha)\} dx}_{E_A(s)} \right] \cdot \left[ \underbrace{\int_{-L/2}^{L/2} f^*(y) \exp\{-jky(S - \sin \alpha)\} dy}_{E_B(s)} \right] dS \\
 &= \int_{-\infty}^{\infty} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} f(x) f^*(y) \exp\{jkS(x-y)\} \cdot \exp\{jk \sin \alpha(x+y)\} dx dy dS
 \end{aligned}$$

적분변수  $S$ 에 대하여 성립하는 관계식인

$$\int_{-\infty}^{\infty} \exp(jsz) ds = \delta(z)$$

를 이용하여 먼저 적분을 행하면

$$\int_{-\infty}^{\infty} \exp\{jkS(x-y)\} dS = \delta(kx-ky) = \delta\{k(x-y)\} .$$

$$\begin{aligned}
 & \therefore \int_{-\infty}^{\infty} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} f(x) f^*(y) \exp\{jkS(x-y)\} \cdot \exp\{jk \sin \alpha(x+y)\} dx dy dS \\
 &= \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} f(x) f^*(y) \exp\{jk \sin \alpha(x+y)\} \cdot \underbrace{\int_{-\infty}^{\infty} \exp\{jkS(x-y)\} dS}_{\delta\{k(x-y)\}} \cdot dx dy .
 \end{aligned}$$

$$= \int_{-L/2}^{L/2} f(x) \cdot \underbrace{\left[ \int_{-L/2}^{L/2} f^*(y) \exp\{jk \sin \alpha (x+y)\} \delta\{k(x-y)\} dy \right]}_{f^*(x) \exp\{jk \sin \alpha \cdot 2x\}} dx$$

$$f^*(x) \exp\{jk \sin \alpha \cdot 2x\} = f^*(x) \exp\{j2kx \sin \alpha\}$$

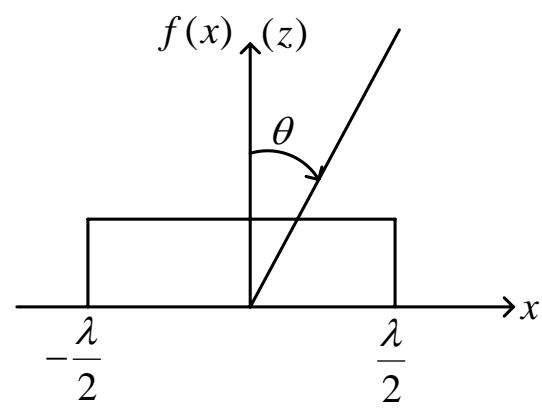
$$= \int_{-L/2}^{L/2} f(x) \cdot f^*(x) \cdot \exp(j2kx \sin \alpha) dx = 0$$



regarded as the radiation pattern corresponding to an aperture distribution equal to the square of the modulus of the basic aperture distribution  $f(x)$ , with  $2\sin \alpha$ , the beam spacing, replacing  $S$

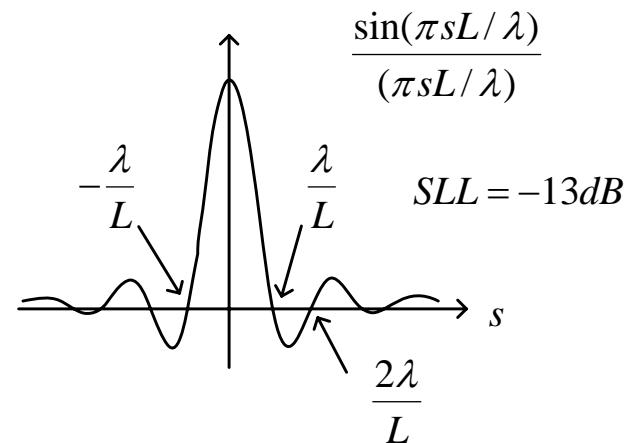
∴ If the spacing of the two beams corresponds to a zero of this radiation pattern, then the two beams are orthogonal.

ex) The case of a uniform distribution [  $f(x) = \text{일정} = 1$  ] over an aperture of length  $L$ .



$$f(x) = 1$$

uniform distribution

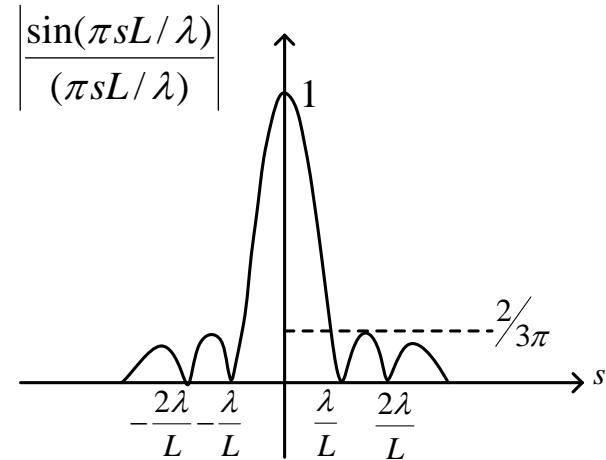


이 경우의 aperture source distribution에  
의한 basic radiation pattern :  $\frac{\sin(\pi SL/\lambda)}{(\pi SL/\lambda)}$ .

→  $\frac{\sin(\pi SL/\lambda)}{(\pi SL/\lambda)}$  에서 first zero는 그림에서와 같아

$S = \sin \theta = \pm \frac{\lambda}{L}$  가 되고 SLL은  $S$  가 대략  $\frac{\lambda}{L} \sim \frac{2\lambda}{L}$  의 중간 지점인  $S = \frac{3\lambda}{2L}$  에서

$$\left| \frac{\sin\left(\pi \cdot \frac{L}{\lambda} \cdot \frac{3\lambda}{2L}\right)}{\pi \cdot \frac{L}{\lambda} \cdot \frac{3\lambda}{2L}} \right| = \left| \frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} \right| = \frac{2}{3\pi} \text{ 이므로,}$$



$$SLL \text{ (sidelobe level)}_{dB} = 20 \log\left(\frac{2}{3\pi}\right) \approx -13[dB]$$

만일  $f(x)=1$ 에  $e^{jkx \sin \alpha}$ 에 해당되는 위상변화를 준다면, 이 때의 aperture source distribution은

$$\underbrace{f(x)e^{jkx \sin \alpha}}_{=1} = f_A ,$$

마찬가지로  $e^{-jkx \sin \alpha}$ 에 해당되는 위상변화를 준다면,

$$\underbrace{f(x)e^{-jkx \sin \alpha}}_{=1} = f_B .$$

이 때,  $f_A$  와  $f_B$ 에 대한 field radiation pattern은 각각,

$$E_A(S) = \int_{-L/2}^{L/2} \underbrace{f(x) \cdot e^{jkx \sin \alpha}}_{f_A} e^{jkxS} dx$$

$$= \int_{-L/2}^{L/2} \underbrace{f(x) \cdot e^{jkx(S + \sin \alpha)}}_{=1} dx ,$$

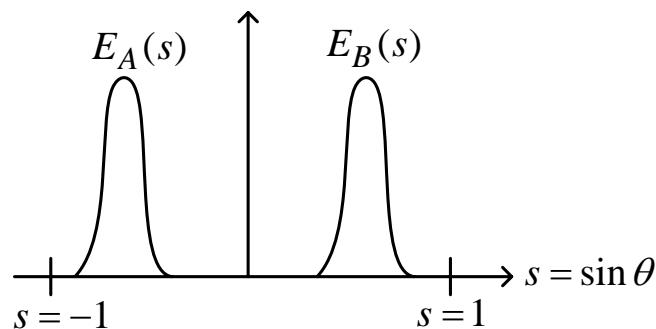
$$E_B(S) = \int_{-L/2}^{L/2} \underbrace{f(x) \cdot e^{-jkx \sin \alpha}}_{f_B} e^{jkxS} dx$$

$$= \int_{-L/2}^{L/2} \underbrace{f(x) \cdot e^{jkx(S - \sin \alpha)}}_{=1} dx ,$$

여기에서 two beams  $E_A(S)$  와  $E_B(S)$ 가 서로 orthogonal 하기 위해서는

$$\int E_A(S) \cdot E_B^*(S) dS = 0 \text{ 를 만족해야 한다.}$$

서로 독립적인 두 빔에 대한 가장 단순한 예 :



$$\int_{-1}^1 E_A(S) \cdot E_B^*(S) dS = 0 .$$

지금 다루고 있는 일반적인 경우에 대하여는

$$\int E_A(S) \cdot E_B^*(S) dS = \int_{-L/2}^{L/2} f(x) f^*(x) \cdot e^{j k x 2 \sin \alpha} dx \text{ 으로 부터}$$

$$\int E_A(S) \cdot E_B^*(S) dS = 0 \text{ 이 되기 위하여는}$$

$$\int_{-L/2}^{L/2} f(x) f^*(x) \cdot e^{j k x 2 \sin \alpha} dx = 0 \text{ 이 되어야 하는데}$$

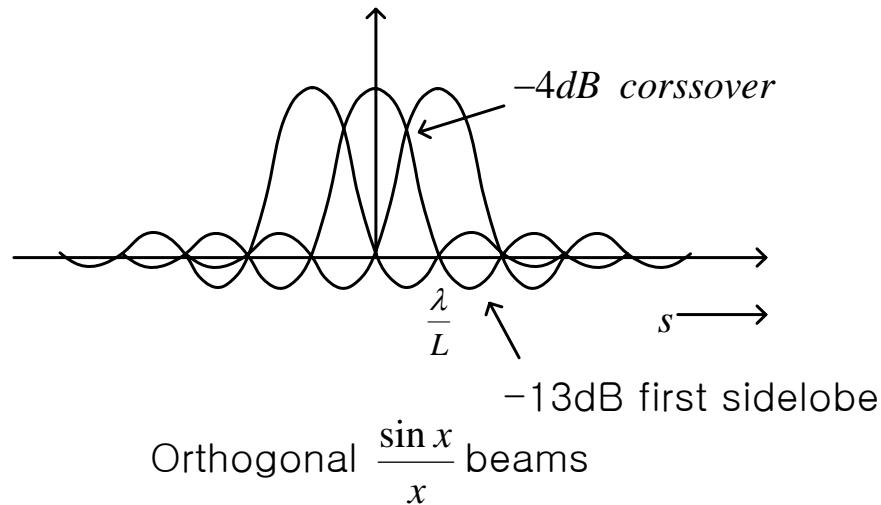
$f(x)=1$  인 경우,

$$\int_{-L/2}^{L/2} e^{j2kx \sin \alpha} dx = \frac{\sin\left(\frac{\pi L}{\lambda} \cdot 2\sin \alpha\right)}{\frac{\pi L}{\lambda} \cdot 2\sin \alpha}$$

로 부터

$\underbrace{2\sin \alpha}_{\text{angular spacing}} = \pm n \frac{\lambda}{L}$  일 때, 즉  $2\sin \alpha = \pm \frac{\lambda}{L}, \pm \frac{2\lambda}{L}, \dots$  일 때,

$\int E_A(S) \cdot E_B^*(S) dS = 0$  가 됨을 볼 수 있다.

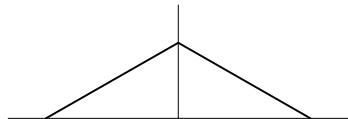
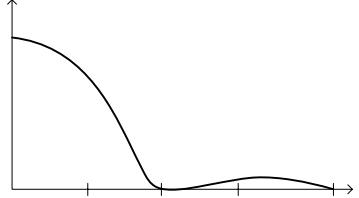
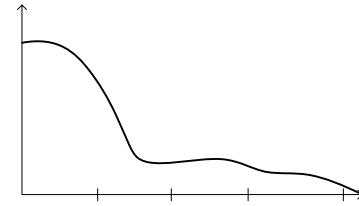


옆의 그림은 angular spacing  $S$  가  
 $S = \pm \frac{\lambda}{L}, \pm \frac{2\lambda}{L}, \dots$  일 때,  $\int E_A(S) E_B^*(S) dS$   
 을 만족하는 서로 orthogonal한 빔들을 도시  
 하고 있다.

note) 한 beam의 peak는 다른 orthogonal beam의 zero에 해당.

Caution) 이는 일반적으로 성립하지는 않는다. ex)  $f(x) = \cos\left(\frac{\pi x}{L}\right)$

Note) 임의의 aperture source distribution에 대하여 orthogonal beam set가 언제나 존재하는 것은 아니다. 아래 그림의 예와 같이 triangular aperture source distribution의 경우, orthogonal beam 관계를 줄 수 있는 beam간의 angular spacing이 존재하지 않는다.

Illumination $f(x)$	Radiation pattern and sidelobe level due to $f(x)$	Angle to first zero	Radiation pattern for $ f(x) ^2$	Spacings and crossover level for orthogonal beams
	$-26dB$ 	$\frac{2\lambda}{L}$	$(\text{No zeros})$ 	$\infty$

Physical explanation :

위의 예에서와 같이, triangular aperture source distribution의 경우, Field radiation pattern  $f(x)$ 의 sidelobe들이 모두 동상(same phase)이므로, 그러한 sidelobe를 갖는 두 빔 패턴의 cross product, 즉  $\int E_A(s) \cdot E_B^*(s) dS$ 는 평균적으로 0이 될 수 없다.  $\therefore \int E_A(s) \cdot E_B^*(s) dS \neq 0$

Multiple beam formers for linear arrays – 3 well-established types

i ) Butler matrix

Ref : J.Butler and R. Lowe, “Beamforming matrix simplifies design of electronically scanned antennas”, Electronics Design, 9, P.170, 1961.

ii ) Maxon matrix.

Ref : R. Hansen (Ed), Microwave scanning antennas, vol. III, Academic press, London, chapter 3. 1966

iii) Rotman lens.

Ref : W. Rotman and R.F. Turner, “ Wide angle lens for line source application ” , IEEE Trans., AP-11, PP. 623–632, 1963

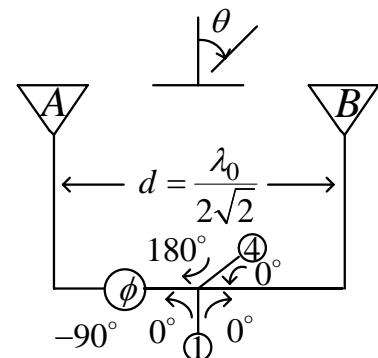
본 강의에서는 대표적인 예로서 Butler matrix에 대하여 살펴보기로 한다.

## 4-5. Butler matrix.

$N$  개의 input ports 중에 하나의 input port를 급전(구동)하여, 그에 상응하는, 복사 각도가 서로 다른  $N$  개의 beam을 복사 할 수 있도록 구성한 feed system을 beam-forming matrices라 하는데, 가장 대표적인 예가 Butler matrix이다.

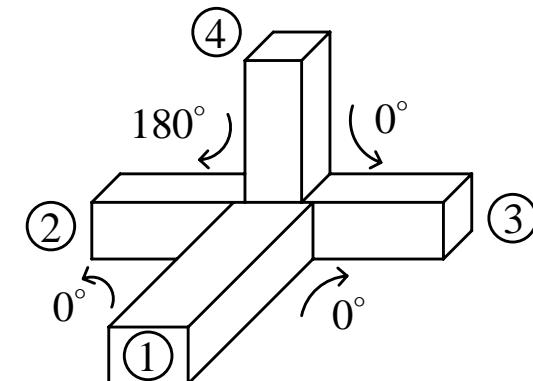
Beam-forming matrices = hybrid junction + fixed phased shifter  
Combination.

ex) two-element array with beam-forming matrix.



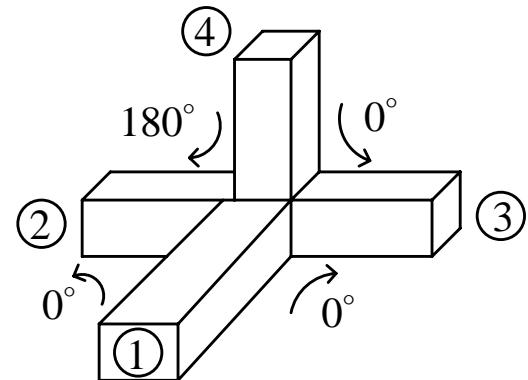
Simple example of Butler  
beam-forming matrix

$\left[ \begin{array}{c} \text{Hybrid junction} \\ +90^\circ \text{phase shifters} \end{array} \right]$

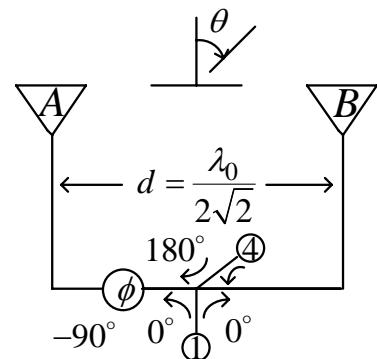


Magic tee hybrid junction.

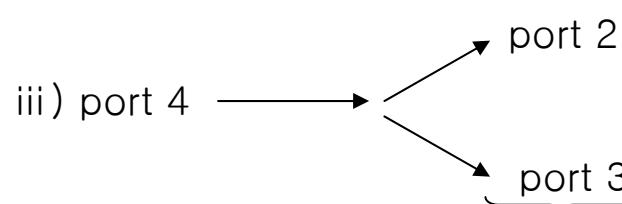
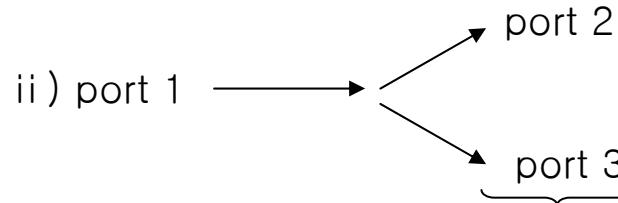
– Properties of the hybrid junction



Hybrid Magic T junction.

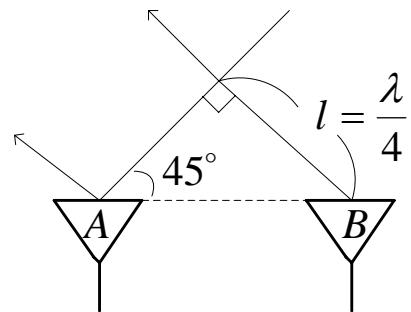


i ) ports (1 and 4, 2 and 3) are uncoupled.

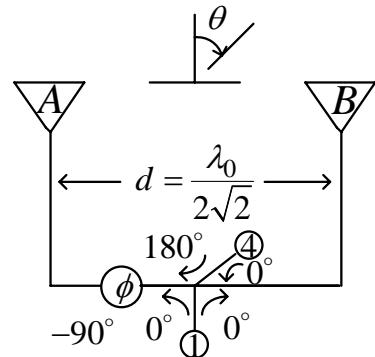


먼저 port 1을 구동한 경우, magic T의 port 2와 3으로 동상으로 분배된 후, 안테나 소자 A로 진행해 가면서  $-90^\circ$ 의 위상변위를 겪으므로, 안테나 소자 A에서의 복사파는 안테나 소자 B에서의 복사파 비하여 위상이  $90^\circ$ 이다.

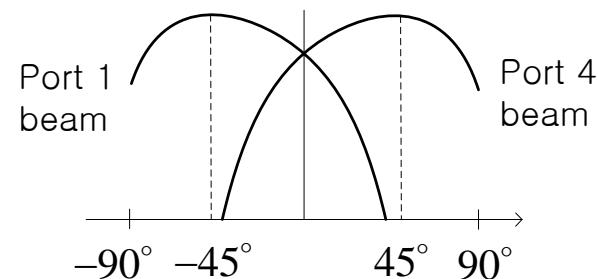
따라서 아래 그림과 같이 왼쪽으로  $45^\circ$  기운 방향으로 복사된다.



; path-length delay  $l$  for maximum beam direction for port 1 beam.



그런데 port 4를 구동하게 되면 magic T의 port 2, 3으로  $180^\circ$  와  $0^\circ$ 의 위상차이가 있어서, 즉 port 2로 결합된 파의 위상이 port 3으로 결합된 파의 위상보다  $180^\circ$  앞서는데, port 2쪽에  $-90^\circ$ 의 위상변위기에 의하여 안테나 소자 A의 위상이 B에 비하여  $90^\circ$ 앞서게 되기 때문에 오른쪽으로  $45^\circ$ 기운 방향으로 복사가 이루어진다.

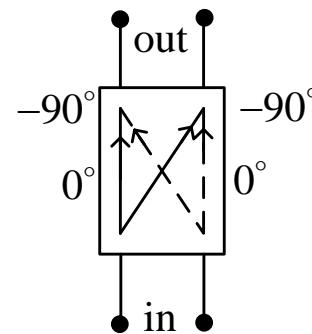
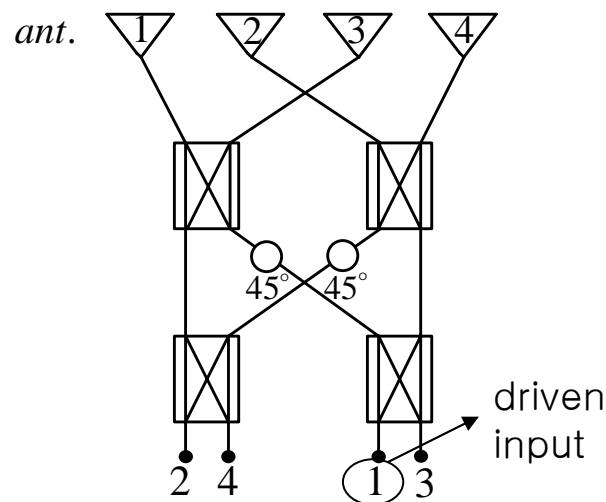
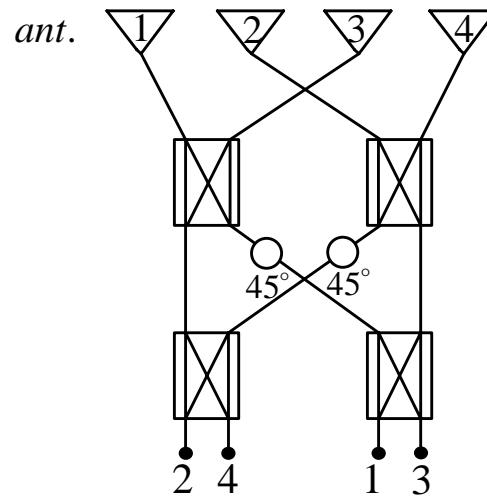


따라서 port 1과 4를 각각 구동한 경우에 형성되는 beam을 도시하면 다음과 같다.

Port 1과 4는 uncoupled 되어 있기 때문에 두 빔은 독립이되어 별도로 동시에 존재한다.

지금까지 다룬 2-element array에 포함된 원리들은 array element의 개수가  $2^N$  ( $N$  : integer)인 모든 array에 적용될 수 있다.

ex) Butler beam-forming matrix for a four-element array.



Transmission phase relationships  
for a  $90^\circ$  phase-lag hybrid  
junction.

- 먼저 port 1을 구동한 경우, 각 array element의 relative aperture phase distribution에 대하여 생각해 보자.

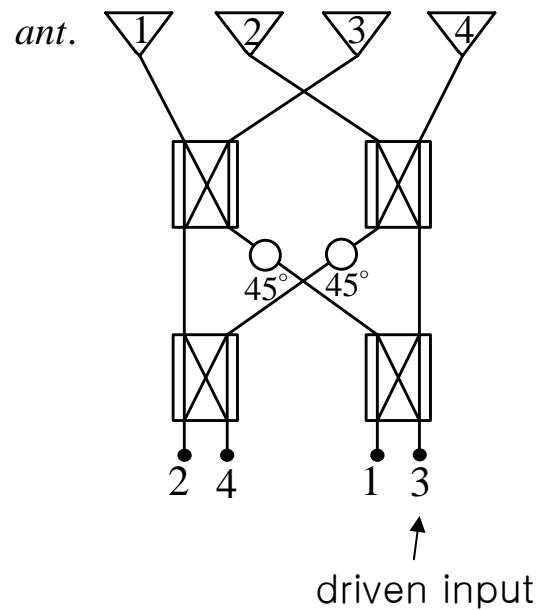
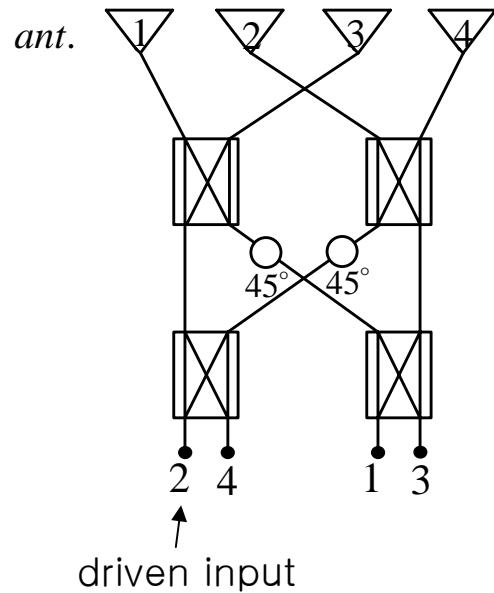
$$\text{Ant. element 1} ; 45^\circ - 90^\circ = \boxed{-45^\circ} \rightarrow 0^\circ$$

$$\text{Ant. element 2} ; -90^\circ - 90^\circ = \boxed{-180^\circ} \rightarrow -135^\circ$$

$$\text{Ant. element 3} ; 45^\circ = \boxed{45^\circ} \rightarrow 90^\circ \rightarrow -270^\circ$$

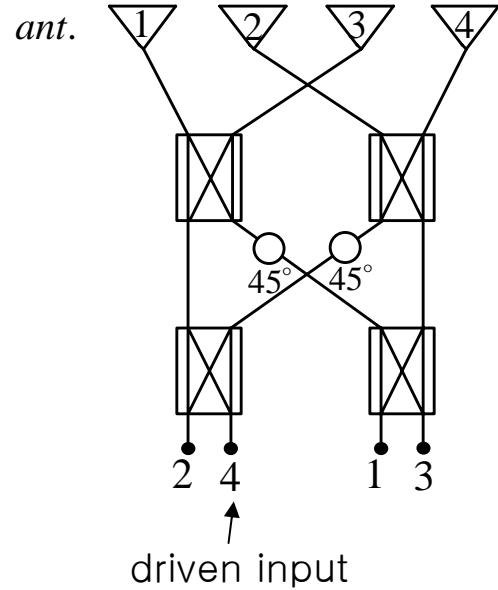
$$\text{Ant. element 4} ; -90^\circ = \boxed{-90^\circ} \rightarrow -45^\circ \rightarrow -405^\circ$$

$$\therefore \text{port 1 구동} ; 0^\circ, -135^\circ, -270^\circ, -405^\circ.$$



Ant. element 1 ;  $0^\circ \rightarrow \boxed{0^\circ} \rightarrow 0^\circ$   
 Ant. element 2 ;  $-90^\circ + 45^\circ = \boxed{-45^\circ} \rightarrow -45^\circ$   
 Ant. element 3 ;  $-90^\circ \rightarrow \boxed{-90^\circ} \rightarrow -90^\circ$   
 Ant. element 4 ;  $-90^\circ + 45^\circ - 90^\circ = \boxed{-135^\circ} \rightarrow -135^\circ$   
 $\therefore$  port 2 구동 ;  $0^\circ, -45^\circ, -90^\circ, -135^\circ$ .

Ant. element 1 ;  $-90^\circ + 45^\circ - 90^\circ = \boxed{-135^\circ} \rightarrow 0^\circ$   
 Ant. element 2 ;  $\boxed{-90^\circ} \rightarrow 45^\circ$   
 Ant. element 3 ;  $-90^\circ + 45^\circ = \boxed{-45^\circ} \rightarrow 90^\circ$   
 Ant. element 4 ;  $\boxed{0^\circ} \rightarrow 135^\circ$   
 $\therefore$  port 3 구동 ;  $0^\circ, 45^\circ, 90^\circ, 135^\circ$ .



Relative aperture phase distribution.

port 1 ;  $0^\circ, -135^\circ, -270^\circ, -405^\circ$

port 2 ;  $0^\circ, -45^\circ, -90^\circ, -135^\circ$

port 3 ;  $0^\circ, 45^\circ, 90^\circ, 135^\circ$

port 4 ;  $0^\circ, 135^\circ, 270^\circ, 405^\circ$

Ant. element 1 ;

$$[-90^\circ] \rightarrow 0^\circ \rightarrow 0^\circ$$

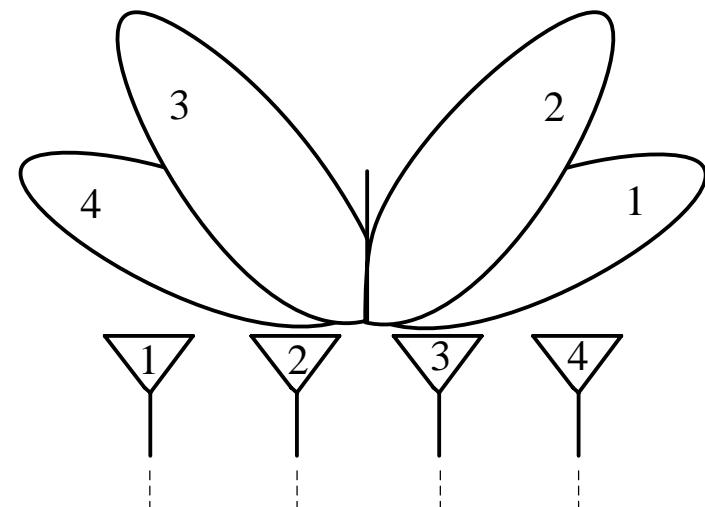
Ant. element 2 ;

$$[45^\circ] \rightarrow 135^\circ \rightarrow 135^\circ$$

Ant. element 3 ;  $-90^\circ - 90^\circ = [-180^\circ] \rightarrow -90^\circ \rightarrow 270^\circ$

Ant. element 4 ;  $45^\circ - 90^\circ = [-45^\circ] \rightarrow 45^\circ \rightarrow 405^\circ$

$\therefore$  port 4 구동 ;  $0^\circ, 135^\circ, 270^\circ, 405^\circ$

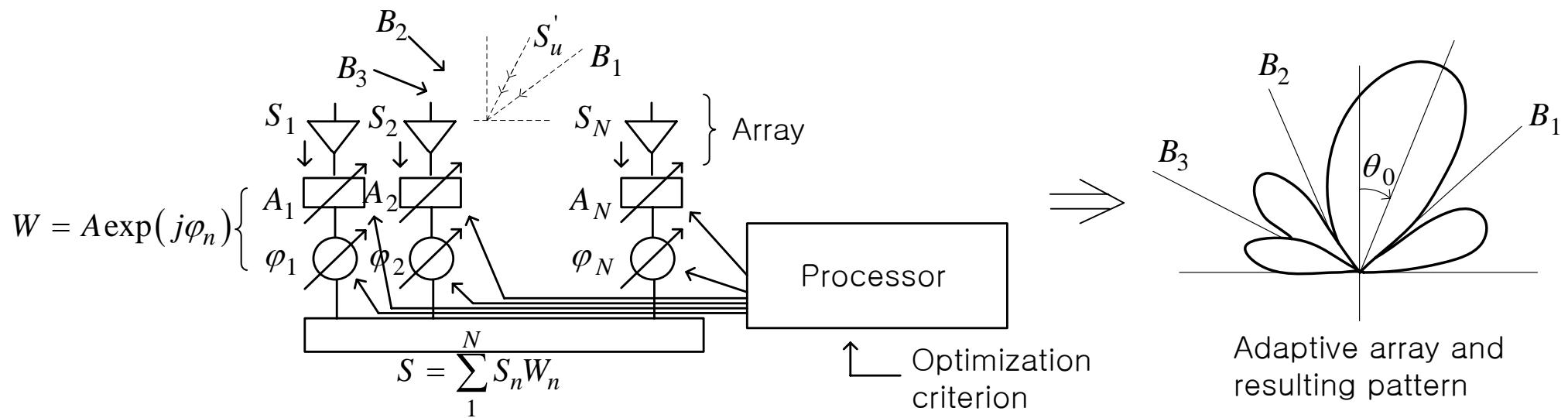


four separate beams produced.

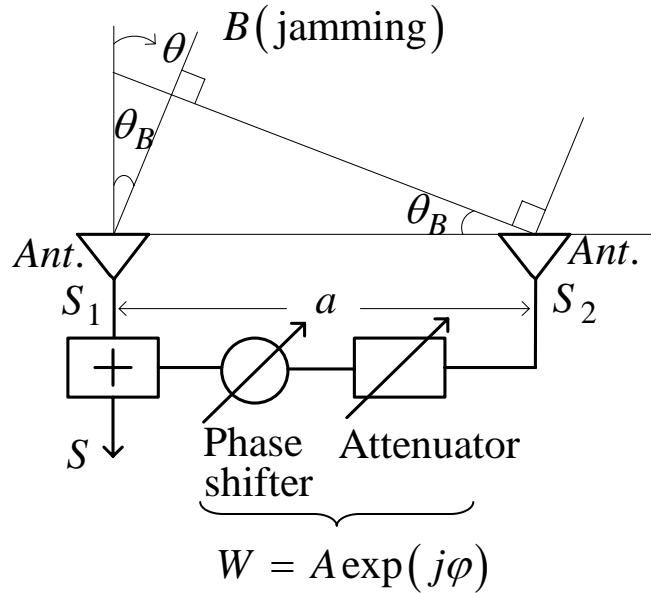
## 4-6. What is adaptive array?

- Adaptive array antenna systems are still currently the subject of intense interest and investigation/development for radar and communications applications.
- The principal reason for the interest is their ability to automatically steer nulls onto undesired source of interference, thereby reducing output noise and enhancing the detection of desired signals.
- These systems usually consist of an array of antenna elements and a real-time adaptive receiver-processor which has feedback control over the elements weights.

아래 그림은 adaptive array의 개념도를 도시하고 있다. 각 antenna element로부터의 수신신호들을 Phase shifter와 attenuator를 사용하여 weighting 과정을 거친 후 summation하게 된다.



Simple example : two-element array



Two-element adaptive array and jammer B.

여기에서 Ant.: omnidirectional antenna.

$S_1$ : signal received by the first antenna element.

$S_2$ : signal received by the second antenna element.

$$\therefore S_2 = S_1 \exp \left[ j \underbrace{\left(2\pi/\lambda\right) a \sin \theta_B}_k \right]$$

이 신호  $S_2$ 가, 위상이  $\varphi$ 이고 감쇄에 상응하는 계수  $A$ 로서 정의되는 계수  $W = Ae^{j\varphi}$ 에 의하여 weighting이 된다면,  $S_B = S_1 + WS_2$

Jammer로부터의 signal  $S_B$  를 상쇄시키려면

$$\begin{aligned} S_B &= S_1 + WS_2 = S_1 + \underbrace{A \exp(j\varphi)}_W \cdot \underbrace{S_1 \exp[j(2\pi a/\lambda) \sin \theta_B]}_{S_2} \\ &= S_1 [1 + A \exp(j\varphi) \cdot \exp[j(2\pi a/\lambda) \sin \theta_B]] = 0 \end{aligned}$$

이로부터

$$\begin{aligned} \hat{A} &= 1, \quad \hat{\phi} = [\pi - (2\pi a/\lambda) \sin \theta_B] \\ \therefore \hat{W} &= \hat{A} \exp(j\hat{\phi}) = e^{j\pi} \cdot \exp[-j(2\pi a/\lambda) \sin \theta_B] = -\exp[-j(2\pi a/\lambda) \sin \theta_B] \end{aligned}$$

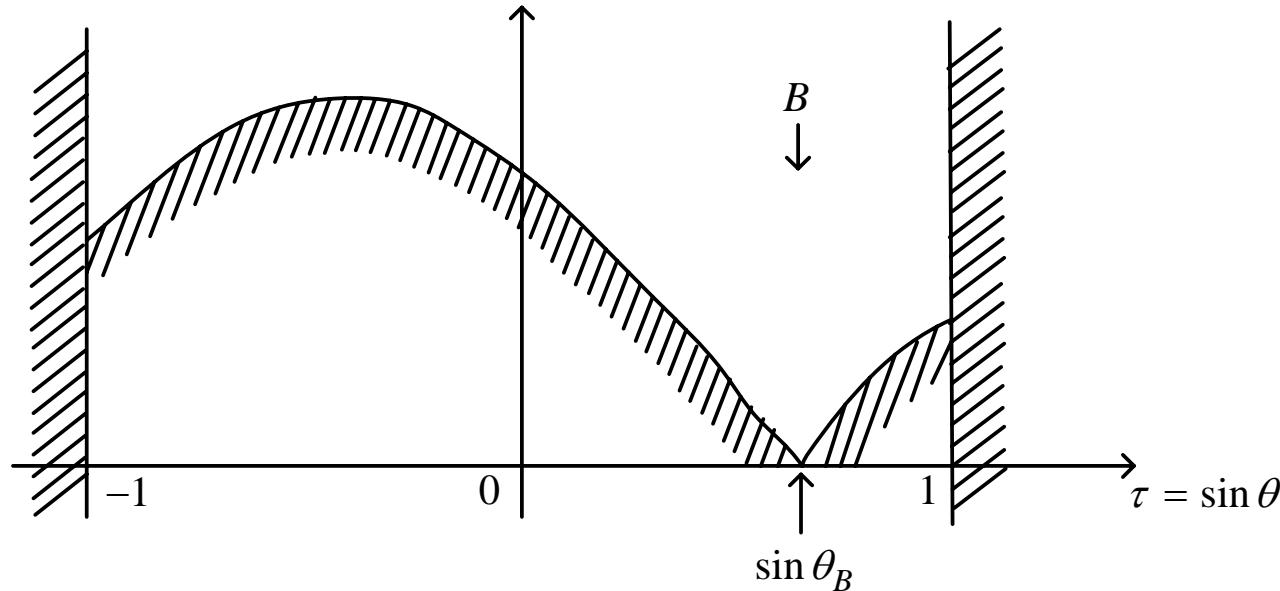
이러한 weighting 과정에 대하여, 임의 각도  $\theta$  로서 입사하는 전파에 대한 수신신호(receiving antenna pattern = radiation pattern)는

$$S(\theta) = S_1 \underbrace{[1 + \hat{W} \exp(j(2\pi a/\lambda) \sin \theta)]}_{F(\theta)}$$

$\therefore \theta$  의 함수로서의 received signal은 다음과 같은 characteristic function  $F(\theta)$ 에 비례,

$$\begin{aligned} F(\theta) &= 1 + \hat{W} \exp[j(2\pi a/\lambda) \sin \theta] = 1 + \underbrace{[-\exp(-j(2\pi a/\lambda) \sin \theta_B) \cdot \exp(j(2\pi a/\lambda) \sin \theta)]}_{\hat{W}} \\ &= 1 - \exp[j(2\pi a/\lambda)(\sin \theta - \sin \theta_B)] \end{aligned}$$

따라서 radiation pattern  $|F(\theta)|$  를 구해보면 아래 그림과 같이 Jammer 방향으로 zero가 된다.



Optimized pattern.

Note)  $F(\theta) = 1 - e^{j\alpha}$ ,  $\alpha = \frac{2\pi a}{\lambda} (\sin \theta - \sin \theta_B)$

$$\begin{aligned} F(\theta) &= e^{j(\alpha/2)} \cdot (e^{-j(\alpha/2)} - e^{j(\alpha/2)}) = e^{j(\alpha/2)} \cdot (-1) \cdot (e^{j(\alpha/2)} - e^{-j(\alpha/2)}) \\ &= e^{j(\alpha/2)} \cdot (-1) \cdot (j2) \sin(\alpha/2) \end{aligned}$$

$$\therefore |F(\theta)| = 2|\sin(\alpha/2)| = 2 \left| \sin \left\{ \frac{\pi a}{\lambda} (\sin \theta - \sin \theta_B) \right\} \right|.$$

## ■ Concluding remarks

- Let's review what we have discussed in Chapter. 4

### 제 4 장. Further Array Topics

4-1. Scanning antennas.

4-2. Case study – Conformal Microstrip phased array for Aircraft test ;

i ) Microstrip array design

ii ) Phase shifter design

4-3. Frequency scanning antenna.

4-4. Orthogonal beams and multiple beam former (MBF).

4-5. Butler Matrix.

4-6. What is adaptive array?